Abstract— To convert the analog signals into samples of
digital signals is called the DSP. In this paper we study the power
spectrum estimation of DSP. There is a fluctuation on DSP
signals thermal noise in resistors & electronic devices.

There are two methods for calculating PSE is parametric &
non parametric methods. In Nonparametric (classical) methods –
begin by estimating the autocorrelation sequence from a given
data. Limitations of non parametric methods are we require
inherent assumptions for autocorrelation estimate.

Parametric method we assume that signal is output of a system
having white noise as an input. We model the system and get its
parameters i.e. coloring filter coefficients and predict the power
spectrum. Here we compare the Yule-Walker method & Burg
method for Power spectrum estimation. In this paper we see that
parametric methods do not need these assumptions.

Keywords— Periodogram, Bartlett method, Welch method
Blackman & Tukey method

I. INTRODUCTION

There are mainly two types of power spectrum estimation
(PSE) methods: parametric and nonparametric. In contrast to
parametric methods, non-parametric methods do not make
any assumptions on the data-generating process or model (e.g.
auto-regressive model). This paper analyzes five common
non-parametric PSE methods. They are: Periodogram
Method, Modified Periodogram Method, Bartlett’s Method,
Welch’s Method, and Blackman-Tukey Method. All these are
estimation methods. The ideal power spectrum of a signal x (n)
can be computed by first finding the ideal autocorrelation r (x)

\[ r_{xx}(\alpha) = E[x(n)x(n+\alpha)] = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(n)x(n+\alpha). \]

This is an average computed over the infinite interval.
However, in image processing, the signal is often of finite
length because the size of any image is finite. Even with a
large image, in order to make the assumption of stationary,
only a small window of the image is considered at a time.
In this paper, it is assumed that the window is small enough so
that the assumption of stationary holds, allowing the
autocorrelation and the power spectrum to be a Fourier
Transform pair. Firstly, the ideal power spectrum is “blurred”
or “smoothened” by the low pass filter, and the amount of
blurring is mainly determined by the width of the main lobe.
Secondly, the power spectrum at any particular frequency is
“leaking” into the side lobes, possibly “masking” the power

II. BARTLETT METHOD

In this method reducing the variance in the periodogram
involves three steps. First the N-point sequence sub divided
into K non overlapping segments. [2]

Where each segment has length M.

\[ x_i(n) = x(n + iM) \]

\[ i = 0, 1, \ldots, K - 1 \]

\[ n = 0, 1, \ldots, M - 1 \]

For each segment, we compute the periodogram,

\[ P_{xx}^{(i)}(f) = \frac{1}{M} \left| \sum_{n=0}^{M-1} x_i(n)e^{-j2\pi fn/M} \right|^2 \]
Finally, we average the periodogram for the K segment to obtain,
\[ P_x^B(f) = \frac{1}{K} \sum_{i=0}^{K-1} P_{xx}^{(i)}(f) \] (3)
The statistical properties of this estimate are easily obtained. First the mean value is,
\[ E[P_x^B(f)] = \frac{1}{K} \sum_{i=0}^{K-1} E[P_{xx}^{(i)}(f)] \] (4)

The value of single periodogram as,
\[ E[P_{xx}^{(i)}(f)] \]
The value of single periodogram as,
\[ E[P_{xx}^{(i)}(f)] = \sum_{m=-(M-1)}^{M-1} \left(1 - \frac{|m|}{M}\right) \gamma_{xx}(m)e^{-j2\pi fm} \] (5)
\[ = \frac{1}{M} \int_{-1/2}^{1/2} \pi_x(\alpha) \left( \frac{\sin \pi(f - \alpha)M}{\sin \pi(f - \alpha)} \right)^2 d\alpha \]

Where
\[ W_B(f) = \frac{1}{M} \left( \frac{\sin \pi fM}{\sin \pi f} \right)^2 \] (6)
The frequency characteristics of Bartlett window,
\[ W_B = \frac{1}{L} M \]
\[ |m| \leq M \] (7)

From equation (5) we observe that the true spectrum is now convolved with the frequency characteristics WB (f) of Bartlett window.

We have to reduced the variance .The variance of Bartlett estimate is,
\[ \text{var}[P_{xx}^B(f)] = \frac{1}{K} \sum_{i=0}^{K-1} \text{var}[P_{xx}^{(i)}] \]
\[ \text{var}[P_{xx}^B(f)] = \frac{1}{K} \text{var}[P_{xx}^{(i)}] \] (8)
We obtain the,
\[ \text{var}[P_{xx}^{(i)}(f)] = \frac{1}{K} \tau_x^2(f) \left[ 1 + \left( \frac{2 \pi fM}{M \sin 2\pi f} \right)^2 \right] \] (9)

Therefore, the variance of Bartlett power spectrum estimate has been reduced by the factor K.

A. Performance characteristics of this Method for PSE

We observe that the Bartlett PSE is asymptotically unbiased & if K is allowed to increase with an increase in N, the estimate is also consistent. [2]

Hence, asymptotically, this estimate is characterized by the quality factor,
\[ Q_B = K = \frac{N}{M} \] (43)

\[ \Delta f = \frac{0.9}{M} \]
\[ Q_B = \frac{N}{0.9/\Delta f} = 1.1N\Delta f \] (44)

III. WELCH METHOD

In this method two basic modification to Bartlett method. First, he allowed the data segments to overlap. Second is to window the data segments prior to computing the periodogram. [1]

The data segments to can be represented as,
\[ x_i(n) = x(n + iD) \]
\[ n = 0,1,......,M-1 \]
\[ i = 0,1,......,L-1 \]

Where iD is starting point for the in sequence.

If D=M Segment do not overlap, L data segments identical to the Number K in Bartlett method.

If D=M/2, 50% overlap.

The modified periodogram is,
\[ \overline{P}_{xx}^{(i)}(f) = \frac{1}{MU} \sum_{n=0}^{M-1} x_i(n)\omega(n)e^{-j2\pi fn} \] (11)

Where, \( i = 0,1, \ldots, L-1 \)

Where U is normalization factor for the power in the window function ,
\[ U = \frac{1}{M} \sum_{n=0}^{M-1} \omega^2(n) \] (12)

Welch PSE is average of these modified periodogram,
\[ P_{xx}^{(w)}(f) = \frac{1}{L} \sum_{i=0}^{L-1} \overline{P}_{xx}^{(i)}(f) \] (13)

The mean value of Welch estimate is
\[ E[P_{xx}^{(w)}(f)] = \frac{1}{L} \sum_{i=0}^{L-1} E[\overline{P}_{xx}^{(i)}(f)] \] (14)

Expected value of modified periodogram is
\[ E[\overline{P}_{xx}^{(i)}(f)] = \frac{1}{MU} \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} \omega(n)\omega(m)E[x_i(n)x_i^*(m)]e^{-j2\pi fn}e^{j2\pi fm} \] (15)
\[ E[\overline{P}_{xx}^{(i)}(f)] = \frac{1}{MU} \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} \omega(n)\omega(m)\gamma_{xx}(n-m)e^{-j2\pi fn}e^{j2\pi fm} \sin ce \]
\[ \gamma_{xx}(n) = \int_{-1/2}^{1/2} \tau_x(\alpha)e^{j2\pi fn} \] (16)

From equation (15) & (16) we get
\[ E[\overline{P}_{xx}^{(i)}(f)] = \frac{1}{MU} \int_{-1/2}^{1/2} \tau_x(\alpha) \left[ \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} \omega(n)\omega(m)e^{j2\pi fn}e^{j2\pi fm} \right] d\alpha \] (17)
\[ E[\overline{P}_{xx}^{(i)}(f)] = \int_{-1/2}^{1/2} \tau_x(\alpha)w(f - \alpha)d\alpha \] (18)

Where by definition
\[ w(f) = \frac{1}{MU} \sum_{n=0}^{M-1} \omega(n)e^{-j2\pi fn} \] (19)
The spectral width of triangular window at the 3-db points is
\[ \Delta f = \frac{1.28}{M} \]  

Quality factor expressed in terms of \( N \& f \),
\[ Q_w = \begin{pmatrix} 0.78N/\Delta f \\ 1.39N/\Delta f \end{pmatrix} \]

IV. BLACKMAN & TUKEY METHOD

In this method, sample the autocorrelation sequence is windowed first & then Fourier transformed to yield the estimate of power spectrum.
\[ P_{xx}^{BT}(f) = \sum_{m=-N/2}^{N/2} \tau_{xx}(m)w(m)e^{-j2\pi mf} \]
Where the window function \( w(n) \) has length 2M-1 & is zero
\[ P_{xx}^{BT}(f) = \int_{-1/2}^{1/2} P_{xx}(\alpha)w(f-\alpha)d\alpha \]  
The window sequence \( w(n) \) should be symmetric (even) about \( m=0 \) to ensure that the estimate of power spectrum is real.
\[ w(f) \geq 0 \quad \text{for} \quad |f| \leq 1/2 \]

Blackman & Tukey PSE,
\[ E[P_{xx}^{BT}(f)] = \int_{-1/2}^{1/2} E[P_{xx}(\alpha)]w(f-\alpha)d\alpha \]  
We have
\[ E[P_{xx}^{BT}(f)] = \int_{-1/2}^{1/2} \tau_{xx}(\alpha)W_{xx}(\alpha-d\theta)d\alpha \]  
From equation 26 & 27 double convolution integral is,
\[ E[P_{xx}^{BT}(f)] = \int_{-1/2}^{1/2} \tau_{xx}(\alpha)W_{xx}(\alpha-d\theta)w(f-\alpha)d\alpha \]  
The working time domain is,
A. Performance characteristics of this Method for PSE

Following equation shows that the value of estimate is asymptotically unbiased.[3]

\[ Q_{BT} = 1.5 \frac{N}{M} \]

\[ \Delta f = \frac{1.28}{2M} = 0.64 \]

\[ Q_{BT} = \frac{1.5}{0.64} N \Delta f = 2.34 N \Delta f \]

We estimate the quality power for rectangular & triangular window.

V. COMPARISON OF THREE METHODS. (FOR QUALITY FACTOR) [1]

<table>
<thead>
<tr>
<th>SR No.</th>
<th>Estimate</th>
<th>Quality Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bartlett Method</td>
<td>1.1N\Delta f</td>
</tr>
<tr>
<td>2</td>
<td>Welch Method</td>
<td>1.39N\Delta f</td>
</tr>
<tr>
<td>3</td>
<td>Blackman &amp; Tukey Method</td>
<td>2.34N\Delta f</td>
</tr>
</tbody>
</table>

A. Points Related to All Methods [1]

1. Due to windowing (leakage frequency due to side lobes) the frequency resolution is low in Bartlett method.
2. Welch Method has got better precision but less frequency resolution than Bartlett method.
3. Blackman-Tukey Method has better variance (even at large lags) and better precision than Bartlett & Welch Method methods. But frequency resolution is less than the Bartlett & Welch method.

VI. CONCLUSION

1. Blackman-Tukey Method has better variance and better precision than Bartlett & Welch Method methods.
2. Bartlett Method has greater frequency resolution
3. Welch Method has better precision
4. Quality factor of Blackman & Tukey more than other two methods

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