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Stochastic Dominance to Study Poverty Measures and its Applications

Nagwa Albehery, Tonghui Wang

Abstract*— Poverty measures are used to measure poverty levels or degrees of poverty in a population. Stochastic dominance refers to a set of relations between distributions. Stochastic dominance orders of poverty measures have been discussed by many authors in the literature. In this paper, definition of stochastic dominance is introduced and used to order poverty measures. Hypotheses testing on stochastic dominance ordering of poverty measures are obtained. For illustration, our results are applied to the real data sets collected in Egypt between 1995/1996 to 2008/2009.*

Index Terms*—Poverty measures, poverty lines, stochastic dominance, Hypotheses testing.*

I. INTRODUCTION

Stochastic dominance has important applications to analyze and to compare poverty levels over time, or between different socioeconomic groups within the same country, or between different countries. Comparing poverty levels are very important in helping decision makers to consider what new plans and economic reforms should be adopted to alleviate these levels in a population. As Duclos (2006) discussed, the easiest way to compare poverty levels is cardinal comparisons by comparing numerical estimates of poverty indices, but choosing single indices may produce contradictory conclusions. Ordinal comparisons or partial orderings are used to solve the cardinal comparisons problem by comparing poverty levels across distributions. In the literature of poverty comparisons, the ordinal poverty comparisons compare poverty levels in two ways: (a) poverty-measure orderings which compares poverty using a class of poverty measures with a fixed poverty line which is a borderline between poor and non-poor people in a population, and (b) poverty-line orderings which compares poverty using a poverty index with a range of poverty lines. Stochastic dominance is related to poverty comparisons which are based on the comparisons across distributions. Suppose we have two distributions A and B with FA(x) and FB(x) respectively, the first order stochastic dominance, the second order stochastic dominance, and the third order stochastic dominance are defined respectively as follows (see Levy (2006) and Linton (2005) for details if interested).

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Definition 1.1 B dominates A in the first order stochastic dominance (FSD), denoted as $B \succeq_1 A$, if and only if

$$F_B(x) \le F_A(x) \quad \text{for all} \quad x \in \Re$$

with

$$F_B(x) < F_A(x) \quad \text{for some} \quad x \in \Re,$$

.

Definition 1.2 B dominates A in the second order stochastic dominance (SSD), denoted as $B \succeq_2 A$, if and only if

$$\int_{-\infty}^{x} [F_B(t) - F_A(t)]dt \le 0 \quad \text{for all} \quad x \in \Re$$

with

$$\int_{-\infty}^{x} [F_B(t) - F_A(t)]dt < 0 \quad \text{for some} \quad x \in \Re,$$

.

Definition 1.3 B dominates A in the third order stochastic dominance (TSD), denoted as $B \succeq_3 A$, if and only if

$$\int_{-\infty}^{y}\int_{-\infty}^{x} [F_B(t) - F_A(t)]dtdx \le 0 \quad \text{for all} \quad y \in \Re$$

with

$$\int_{-\infty}^{y}\int_{-\infty}^{x} [F_B(t) - F_A(t)]dtdx < 0 \quad \text{for some} \quad y \in \Re,$$

.

In this paper, stochastic dominance for poverty orderings is introduced in Section 2. Foster poverty orderings using stochastic dominance approach is discussed in Section 3. Hypotheses testing for first, second, and third order stochastic dominance on Foster poverty measures are obtained in Section 4. In Section 5, our main results are applied to the real survey data on household expenditures collected in Egypt (from Egyptian Central Agency of Statistics) between 1995/1996 and 2008/2009, as an illustration.

II. STOCHASTIC DOMINANCE FOR POVERTY ORDERINGS

Foster (1988) derived the partial orderings of discrete distributions from some poverty indices and sets of welfare functions. Also, the partial poverty orderings is defined by: (a) finite-population income distribution defined over positive income values X has less poverty than a finite-population income distribution defined over positive income values Y with respect to the poverty index P, if and only if

$$P(x,z) \le P(y,z) \quad \text{for all} \quad z \in \Re_+,$$

and

$$P(x,z) < P(y,z) \quad \text{for some} \quad z \in \Re_+$$

where z is a poverty line.

Davidson (2000) said Distribution B dominates distribution A at order s, up to poverty line z, if

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$$D_B^s(x) \le D_A^s(x) \quad \text{for all} \quad x \le z,$$

with

$$D_B^s(x) < D_A^s(x) \quad \text{for some} \quad x \le z,$$

where

$$D_i^s(x) = \int_0^x D_i^{(s-1)}(t)dt, \quad i = A, B,$$

and

$$D^s(x) = \frac{1}{(s-1)!}\int_0^x (x-t)^{s-1}dF(t).$$

In this paper, poverty-line orderings are used to compare poverty levels using ordinal comparisons. Poverty-line orderings are defined by comparing poverty levels, using a poverty index with a range of poverty lines. We will use Foster's indices with a range of poverty line z ϵ [z min, z max], where (z min) is the minimum value of an estimated relative poverty line and, (z max), is the maximum value of an estimated relative poverty line. The relative poverty line defined by a percentage of mean or median income or expenditure is used as the borderline between the poor and non-poor people see Weisbrod (1965), Oti (1990) and Zheng (2001). This line is estimated as a point in the distribution of income or expenditure, then it is updated automatically over time, when the income or expenditure changes. For this reason, the relative poverty line is considered the common poverty line that is used in the literature of ordinal poverty orderings.

III. FOSTER POVERTY ORDERINGS USING STOCHASTIC DOMINANCE

Foster poverty measure defined by

$$P_\alpha(F,z) = \int_0^{+\infty} I_{[0,z)}(x)\left(1-\frac{x}{z}\right)^\alpha dF(x), \quad \alpha \ge 0, \tag{1}$$

will be used in this section to compare the poverty levels using stochastic dominance. Let $F_0(x) = F(x)$ be the c.d.f. of X. Let $z \in [z_-, z_+]$ where z is the relative poverty line estimated between the minimum value and the maximum value. At $\alpha = 0$, the Foster poverty measure $P_0(F, z)$ is the head count ratio H, then we define the first order stochastic dominance using the head count ratio index as follows (i) The head count ratio in population B, is less than the head count ratio in population A; or, B dominates A at first order stochastic dominance, if and only if

$$F_B(x_B) \le F_A(x_A) \quad \text{for all} \quad x_i \in [z_{i-}, z_{i+}], \quad i = A, B,$$

and

$$F_B(x_B) < F_A(x_A) \quad \text{for some} \quad x_i \in [z_{i-}, z_{i+}], \quad i = A, B.$$

At $\alpha = 1$, the Foster poverty measure $P_1(F, z)$ is the poverty gap ratio defined by

$$P_1(F,z) = \int_0^{+\infty} I_{[0,z)}(x)\left(1-\frac{x}{z}\right)dF(x),$$

then,

$$zP_1(F,z) = \int_0^z (z-x)\,dF(x),$$

integrating by parts see Foster (1984), we conclude that

$$zP_1(F,z) = \int_0^z F(x)dx.$$

Let $F_1(z) = \int_0^z F_0(t)dt = \int_0^z F(t)dt$, then

$$zP_1(F,z) = F_1(z). \tag{2}$$

Now, we define the second order stochastic dominance using the poverty gap ratio as follows

(ii) The poverty gap ratio in B, is less than the poverty gap ratio in A; or, B dominates A at second order stochastic dominance, if and only

$$z_B P_1(F_B, x_B) \le z_A P_1(F_A, x_A) \quad \text{for all} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B, \tag{3}$$

and

$$z_B P_1(F_B, x_B) < z_A P_1(F_A, x_A) \quad \text{for some} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B. \tag{4}$$

At $\alpha = 2$, the Foster poverty measure $P_2(F, z)$ is the severity of poverty defined by

$$P_2(F,z) = \int_0^{+\infty} I_{[0,z)}(x)\left(1-\frac{x}{z}\right)^2 dF(x),$$

then,

$$z^2 P_2(F,z) = \int_0^z (z-x)^2\, dF(x),$$

integrating by parts, we conclude that

$$\frac{z^2}{2}P_2(F,z) = F_2(z),$$

where $F_2(z) = \int_0^z F_1(t)dt = \int_0^z \int_0^z F(t)dtdt$, then

$$\frac{z^2}{2}P_2(F,z) = F_2(z). \tag{5}$$

Now, we define the third order stochastic dominance using the severity of poverty as follows

(iii) The severity of poverty in B, is less than the severity of poverty in A; or, B dominates A at third order stochastic dominance, if and only if is

$$z_B^2 P_2(F_B, x_B) \le z_A^2 P_2(F_A, x_A) \quad \text{for all} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B,$$

and

$$z_B^2 P_2(F_B, x_B) < z_A^2 P_2(F_A, x_A) \quad \text{for some} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B.$$

Now, we want to test the first, second, and third order stochastic dominance.

IV. HYPOTHESES TESTING FOR FIRST, SECOND, AND THIRD ORDER STOCHASTIC DOMINANCE ON FOSTER POVERTY MEASURES

Foster's indices which are given in (1) are estimated using the results from U-statistics. Also, the asymptotic distributions of the estimators of Foster's indices are derived see, Xu (2007) and Albehery (2011). In this work, the advantages of the estimation and the asymptotic distributions of the estimators of Foster's indices will be used to test if the first, second, and third order stochastic dominance are significant or insignificant. Also, the intersection-union principle will be applied to construct the test statistics, see Kaur (1994), and Davidson (2000).

A. Testing first-order stochastic dominance

Let the hypothesis be

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$$H_0: F_A(x_A) \le F_B(x_B) \quad \text{for all} \quad x_i \in [z_{i-}, z_{i+}], \quad i = A, B,$$

against

$$H_1: F_A(x_A) > F_B(x_B) \quad \text{for some} \quad x_i \in [z_{i-}, z_{i+}], \quad i = A, B.$$

We know that $\hat{F}_i(x_i) = \hat{H}_i, \quad i = A, B$. Let

$$z^{(1)}(x_A, x_B) = \frac{\hat{F}_A(x_A) - \hat{F}_B(x_B)}{\sqrt{\frac{\hat{\sigma}_{11_A}}{n_A} + \frac{\hat{\sigma}_{11_B}}{n_B}}}, \quad x_i \in [z_{i-}, z_{i+}], \quad i = A, B,$$

where $\hat{\sigma}_{11}$ is the estimator σ_{11}, and $\sigma_{11} = Var\left(\sqrt{n}(\hat{F}_i(x_i) - H)\right)$, n_A and n_B are the sample size from populations A and B respectively. The test statistic will be

$$k_1 = \inf z^{(1)}(x_A, x_B), \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B.$$

Let z_α be the $(1-\alpha)$ percentile of standard normal distribution, then we reject H_0 if and only if $k_1 > z_\alpha$, where $z^{(1)}(x_A, x_B)$ follows asymptotic normal distribution with zero mean and unit variance.

B. Testing second-order stochastic dominance

The hypothesis is

$$H_0: z_A P_1(F_A, x_A) \le z_B P_1(F_B, x_B) \quad \text{for all} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B,$$

against

$$H_1: z_A P_1(F_A, x_A) > z_B P_1(F_B, x_B) \quad \text{for some} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B.$$

Let

$$z^{(2)}(x_A, x_B) = \frac{z_A \hat{P}_1(F_A, x_A) - z_B \hat{P}_1(F_B, x_B)}{\sqrt{\frac{z_A^2 \hat{\sigma}_{22_A}}{n_A} + \frac{z_B^2 \hat{\sigma}_{22_B}}{n_B}}}, \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, i = A, B,$$

where $\hat{\sigma}_{22}$ is the estimator of σ_{22}, and $\sigma_{22} = Var\left(\sqrt{n}(\hat{P}_1(F_i, x_i) - P_1(F_i, x_i))\right)$.
The test statistic will be

$$k_2 = \inf z^{(2)}(x_A, x_B), \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B,$$

which is $z^{(2)}(x_A, x_B)$ follows asymptotic normal distribution with zero mean and unit variance.

C. Testing third-order stochastic dominance

The hypothesis is

$$H_0: z_A^2 P_2(F_A, x_A) \le z_B^2 P_2(F_B, x_B) \quad \text{for all} \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B,$$

against

$$H_1: z_A^2 P_2(F_A, x_A) > z_B^2 P_2(F_B, x_B) \quad \text{for some} \quad z_i \in [z_i-, zi_+], \quad x_i < z_i, i = A, B.$$

Let

$$z^{(3)}(x_A, x_B) = \frac{z_A^2 \hat{P}_2(F_A, x_A) - z_B^2 \hat{P}_2(F_B, x_B)}{\sqrt{\frac{z_A^4 \hat{\sigma}_{33_A}}{n_A} + \frac{z_B^4 \hat{\sigma}_{33_B}}{n_B}}}, \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, i = A, B,$$

where $\hat{\sigma}_{33}$ is the estimator of σ_{33}, and $\sigma_{33} = Var\left(\sqrt{n}(\hat{P}_2(F_i, x_i) - P_2(F_i, x_i))\right)$. The test statistic will be

$$k_3 = \inf z^{(3)}(x_A, x_B), \quad z_i \in [z_{i-}, z_{i+}], \quad x_i < z_i, \quad i = A, B,$$

which is $z^{(3)}(x_A, x_B)$ follows asymptotic normal distribution with zero mean and unit variance.

V. REAL DATA APPLICATIONS

The most important data sources to measure the poverty levels in Egypt are household surveys. Egyptian household surveys are available for the years 1995/1996, 1999/2000, 2004/2005 and 2008/2009. Each period is written as two years because each survey starts in mid-year of the first period and ends in mid-year of the second one. The title of the surveys for 1995/1996, 1999/2000, 2004/2005 and 2008/2009 was "The Research of Consumption and Expenditure in Egypt" and all households surveys are produced by the Egyptian Central Agency of Statistics (ECAS). In this paper, we will use household expenditure instead of income because many reported incomes in developing countries might be far less than real incomes. Also, in the developing countries, the income data is limited since many people don't report secondary sources of income. In our work, consumption expenditure data sets on urban areas and on rural areas are collected independently. Also for each consumption expenditure group the average expenditure, the average expenditure on food, the number of households NH, and the number of individuals NI are given. The values of estimated Foster poverty indices using rural, urban, and total data collected in Egypt from 1995/1996 to 2008/2009 at the estimated values of relative poverty lines are listed in Tables 2, 3, 4, 5 and Table 6, respectively. The values of relative poverty lines are estimated by

(1) $z_1 = 0.2$ of the median of annual household expenditure.
(2) $z_2 = 0.3$ of the median of annual household expenditure.
(3) $z_3 = 0.4$ of the median of annual household expenditure.
(4) $z_4 = 0.5$ of the median of annual household expenditure.
(5) $z_5 = 0.6$ of the median of annual household expenditure.
(6) $z_6 = 0.7$ of the median of annual household expenditure.
(7) $z_7 = 0.8$ of the median of annual household expenditure.

Where the estimated median of annual household expenditure are listed in Table1. Also, the values of test statistic K2 and k3 for testing second and third order stochastic dominance problems for Foster poverty measures given in subsection 4.1, 4.2, and 4.3 are listed in Table 7. From the Table 7, at 5% significant level, urban survey data in 2004/2005 is significantly stochastically dominated urban survey data in 1999/2000 with respect to the poverty gap ratio and the severity of poverty. Also, rural, urban, and total survey data in 2008/2009 are significantly stochastically dominated rural, urban, and total survey data in 2004/2005, respectively with respect to the poverty gap ratio and the severity of poverty. In addition, the figures that show the second and the third order stochastic dominance for Foster poverty measures are displayed in Figure 1 to Figure 10.

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A. Tables

Table 1: Estimated median annual household expenditure using rural, urban, and total survey data from 95/96 to 08/09.

	1995-1996	1999-2000	2004-2005	2008-2009
Urban	6183.5	8970.3	10733.1	15975.1
Rural	4804.3	6487.5	7993.8	13113.9
Total	...	...	8993.3	13985.4

Table 2: Estimated Foster poverty indices using a range of relative poverty line and rural survey data in 95/96 and 99/00.

	Rural 1995/1996						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	960.86	1441.3	1921.7	2420.2	2882.6	3363.0	3843.5
$\hat{H}$	0.94%	3.51%	3.51 %	8.95 %	17.96%	17.96 %	30.03 %
$\hat{P}_1$	0.20%	0.62 %	1.34 %	2.64 %	3.83%	5.85%	8.09%
$\hat{P}_2$	0.04%	0.24%	0.59 %	1.08 %	1.74%	2.58%	3.58 %
	Rural 1999/2000						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	1297.5	1946.3	2595.0	3243.8	3892.5	4511.3	5190.0
$\hat{H}$	0.12%	1.88%	5.99 %	5.99 %	13.13%	24.65 %	24.65 %
$\hat{P}_1$	0.04%	0.40 %	0.81 %	1.84 %	3.18%	4.62%	7.12%
$\hat{P}_2$	0.01%	0.10%	0.33 %	0.71 %	1.23%	1.95%	2.89 %

Table 3: Estimated Foster poverty indices using a range of relative poverty line and rural survey data in 04/05 and 08/09.

	Rural 2004/2005						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	1598.7	2398.1	3197.5	3996.9	4796.3	5595.7	6395.0
$\hat{H}$	0.82%	0.82%	2.79 %	6.70 %	13.3%	22.5 %	22.5 %
$\hat{P}_1$	0.01%	0.28%	0.81 %	1.64 %	2.82%	4.42%	6.68%
$\hat{P}_2$	0.00%	0.10%	0.29 %	0.60 %	1.08%	1.75%	2.66 %
	Rural 2008/2009						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	2622.8	3934.2	5245.2	6556.9	7868.3	9179.7	10491.1
$\hat{H}$	0.74%	1.71%	2.97 %	7.71 %	12.03%	17.09 %	23.12 %
$\hat{P}_1$	0.10%	0.41 %	0.91 %	1.63 %	2.84%	4.51%	6.66%
$\hat{P}_2$	0.01%	0.15%	0.37 %	0.70 %	1.16%	1.82%	2.70 %

Table 4: Estimated Foster poverty indices using a range of relative poverty line and urban survey data in 95/96 and 99/00.

	Urban 1995/1996						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	1236.7	1855.0	2473.4	3091.7	3710.1	4328.4	4946.8
$\hat{H}$	0.56%	1.28%	5.32 %	11.27 %	19.06%	19.06 %	29.05 %
$\hat{P}_1$	0.10%	0.42 %	1.34 %	2.64 %	4.31%	6.42%	9.07%
$\hat{P}_2$	0.03%	0.16%	0.45 %	0.97 %	1.73%	2.71%	3.90 %
	Urban 1999/2000						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	1794.1	2691.1	3588.1	4485.2	5382.2	6279.1	7176.2
$\hat{H}$	0.77%	2.67%	6.35 %	6.35 %	12.41%	20.62 %	29.71 %
$\hat{P}_1$	0.07%	0.40 %	1.03 %	2.09 %	3.75%	5.99%	8.67%
$\hat{P}_2$	0.01%	0.13%	0.39 %	0.83 %	1.49%	2.39%	3.54 %

Table 5: Estimated Foster poverty indices using a range of relative poverty line and urban survey data in 04/05 and 08/09.

	Urban 2004/2005						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	2146.6	3219.9	4293.2	5366.6	6439.9	7513.2	8586.5
$\hat{H}$	0.34%	1.28%	3.37 %	6.64 %	11.89%	18.64 %	35.01 %
$\hat{P}_1$	0.10%	0.37 %	0.96 %	1.95 %	3.45%	5.56%	8.23%
$\hat{P}_2$	0.03%	0.13%	0.36 %	0.75 %	1.35%	2.20%	3.31 %
	Urban 2008/2009						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	3195.0	4792.5	6390.0	7987.6	9585.1	11182.6	12780.1
$\hat{H}$	0.39%	1.57%	2.74 %	6.75 %	13.93%	21.25 %	21.25 %
$\hat{P}_1$	0.10%	0.37 %	0.82 %	1.63 %	2.87%	4.72%	6.79%
$\hat{P}_2$	0.04%	0.15%	0.34 %	0.65 %	1.13%	1.83%	2.76 %

Table 6: Estimated Foster poverty indices using a range of relative poverty line and total survey data in 04/05 and 08/09.

	Total 2004/2005						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	1798.7	2698.0	3597.3	4496.7	5396.0	6295.3	7194.7
$\hat{H}$	0.60%	2.09%	5.15 %	5.15 %	10.16%	17.56 %	26.63 %
$\hat{P}_1$	0.08%	0.33 %	0.80 %	1.67 %	3.05%	4.96%	7.39%
$\hat{P}_2$	0.01%	0.11%	0.31 %	0.66 %	1.19%	1.94%	2.92 %
	Total 2008/2009						
	z_1	z_2	z_3	z_4	z_5	z_6	z_7
measures	2797.1	4195.6	5594.2	6992.7	8391.2	9789.8	11188.3
$\hat{H}$	0.58%	1.32%	3.94 %	6.16 %	9.64%	18.94 %	28.23 %
$\hat{P}_1$	0.11%	0.38 %	0.83 %	1.59 %	2.72%	4.40%	6.57%
$\hat{P}_2$	0.04%	0.15%	0.35 %	0.65 %	1.11%	1.75%	2.61 %

Table 7: The values of test statistic k_2 and k_3 for testing second and third order stochastic dominance on Foster poverty measures.

	A: 2004/2005, B: 1999/2000	
Area	Test statistic value k_2	Test statistic value k_3
Rural Egypt (A) Rural Egypt (B)	0.0001	0.681
Urban Egypt (A) Urban Egypt (B)	3.175	6.242
	A: 2008/2009, B: 2004/2005	
Area	Test statistic k_2	Test statistic k_3
Rural Egypt (A) Rural Egypt (B)	7.750	7.460
Urban Egypt (A) Urban Egypt (B)	2.940	4.310
Total Egypt (A) Total Egypt (B)	6.473	8.554

B. Figures

z/Median	Median*P1 Rural (1999/2000)	Median*P1 Rural (2004/2005)
0.2	2.85	0.81
0.3	26.21	22.51
0.4	52.52	64.69
0.5	119.68	130.71
0.6	206.02	225.77
0.7	299.68	352.97
0.8	462.09	533.99

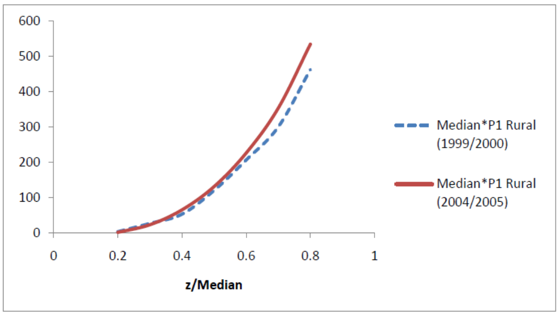

Figure 1: Second order stochastic dominance - Rural 99/00, Rural 04/05.

This figure displays the second order stochastic dominance between rural data collected in 1999/2000 and in 2004/2005. It shows that the rural 1999/2000 stochastically dominated rural 2004/2005 with respect to the poverty gap ratio and using the relative poverty line, i.e., the poverty gap ratio for rural 1999/2000 is less than the poverty gap ratio for rural 2004/2005.

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z/Median	z*Median*P2 Rural (1999/2000)	z*Median*P2 Rural (2004/2005)
0.2	1375.35	15.72
0.3	13117.73	18441.7
0.4	55143.75	73926.66
0.5	149017.88	192650.58
0.6	310037.63	412959.71
0.7	573877.76	784791.32
0.8	972035.1	1360480.8

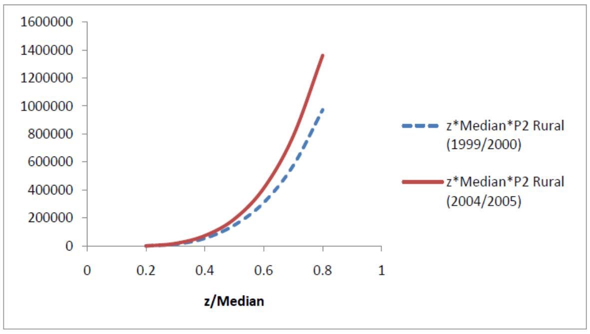

Figure 2: Third order stochastic dominance - Rural 99/00, Rural 04/05.

This figure displays the third order stochastic dominance between rural data collected in 1999/2000 and in 2004/2005. It shows that the rural 1999/2000 stochastically dominated rural 2004/2005 with respect to the severity of poverty and using the relative poverty line, i.e., the severity of poverty for rural 1999/2000 is less than the severity of poverty for rural 2004/2005.

z/Median	Median*P1 Rural (2004/2005)	Median*P1 Rural (2008/2009)
0.2	0.81	13.3
0.3	22.51	54.03
0.4	64.69	119.68
0.5	130.71	214.33
0.6	225.77	372.22
0.7	352.97	591.1
0.8	533.99	872.83

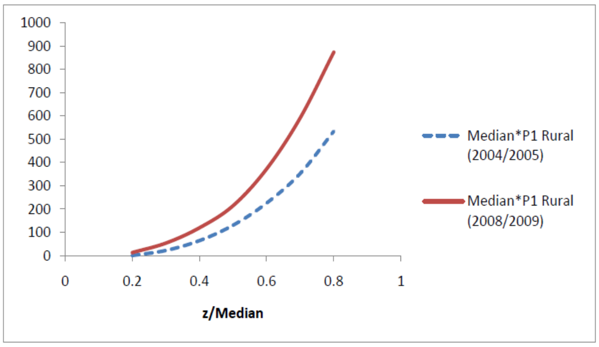

Figure 3: Second order stochastic dominance - Rural 04/05, Rural 08/09.

This figure displays the second order stochastic dominance between rural data collected in 2004/2005 and in 2008/2009. It shows that the rural 2004/2005 stochastically dominated rural 2008/2009 with respect to the poverty gap ratio and using the relative poverty line, i.e., the poverty gap ratio for rural 2004/2005 is less than the poverty gap ratio for rural 2008/2009.

z/Median	z*Median*P2 Rural (2004/2005)	z*Median*P2 Rural (2008/2009)
0.2	15.78	13323.72
0.3	18437.25	79470.23
0.4	73926.66	256402.97
0.5	192650.58	597665.99
0.6	412959.71	1196459.78
0.7	784791.32	2187988.65
0.8	1360480.81	3713903.53

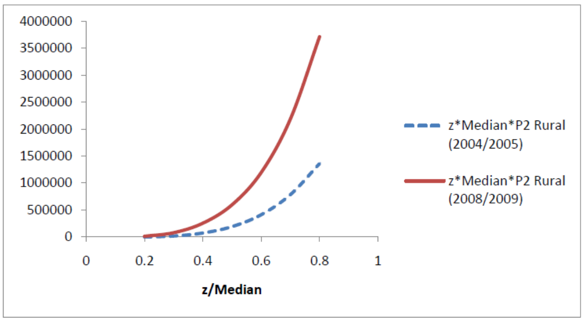

Figure 4: Third order stochastic dominance - Rural 04/05, Rural 08/09.

This figure displays the third order stochastic dominance between rural data collected in 2004/2005 and in 2008/2009. It shows that the rural 2004/2005 stochastically dominated rural 2008/2009 with respect to the severity of poverty and using the relative poverty line, i.e., the severity of poverty for rural 2004/2005 is less than the severity of poverty for rural 2008/2009.

z/Median	Median *P1 Urban (1999/2000)	Median *P1 Urban (2004/2005)
0.2	6.4	10.6
0.3	35.69	39.48
0.4	92.3	102.83
0.5	187.29	209.36
0.6	336.77	370.1
0.7	537.2	596.6
0.8	777.43	883.16

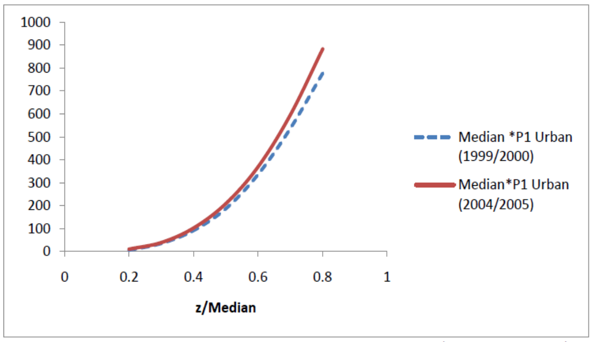

Figure 5: Second order stochastic dominance - Urban 99/00, Urban 04/05.

This figure displays the second order stochastic dominance between urban data collected in 1999/2000 and in 2004/2005. It shows that the urban 1999/2000 stochastically dominated urban 2004/2005 with respect to the poverty gap ratio and using the relative poverty line, i.e., the poverty gap ratio for urban 1999/2000 is less than the poverty gap ratio for urban 2004/2005.

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z/Median	z*Median*P2 Urban (1999/2000)	z*Median*P2 Urban (2004/2005)
0.2	1478.89	6590.12
0.3	3034.83	45755.21
0.4	126460.81	163572.44
0.5	335906.49	431041.3
0.6	718016.73	933006.92
0.7	1344229.48	1774385.36
0.8	2276680.8	3052150.18

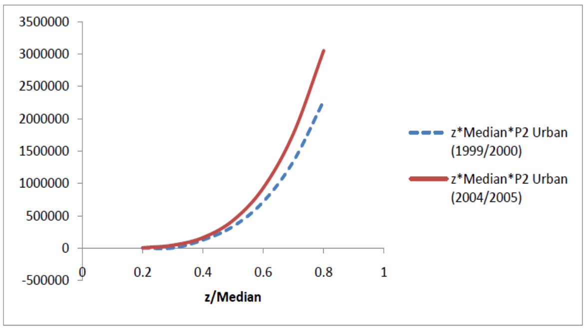

Figure 6: Third order stochastic dominance - Urban 99/00, Urban 04/05.

This figure displays the third order stochastic dominance between urban data collected in 1999/2000 and in 2004/2005. It shows that the urban 1999/2000 stochastically dominated urban 2004/2005 with respect to the severity of poverty and using the relative poverty line, i.e., the severity of poverty for urban 1999/2000 is less than the severity of poverty for urban 2004/2005.

z/Median	Median*P1 Urban (2004/2005)	Median*P1 Urban (2008/2009)
0.2	10.6	18.04
0.3	39.48	58.37
0.4	102.83	131.09
0.5	209.36	260.15
0.6	370.1	458.31
0.7	596.6	754.6
0.8	883.16	1084.57

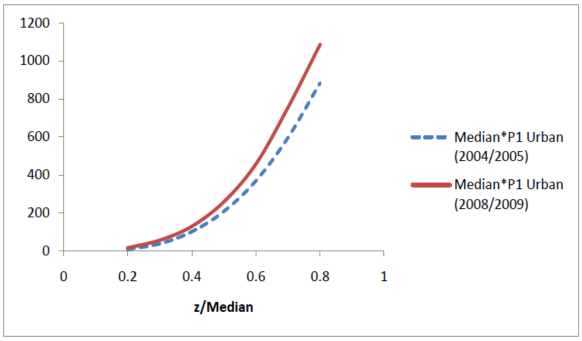

Figure 7: Second order stochastic dominance - Urban 04/05, Urban 08/09.

This figure displays the second order stochastic dominance between urban data collected in 2004/2005 and in 2008/2009. It shows that the urban 2004/2005 stochastically dominated urban 2008/2009 with respect to the poverty gap ratio and using the relative poverty line, i.e., the poverty gap ratio for urban 2004/2005 is less than the poverty gap ratio for urban 2008/2009.

z/Median	z*Median*P2 Urban (2004/2005)	z*Median*P2 Urban (2008/2009)
0.2	6590.12	20927.38
0.3	45755.21	111618.02
0.4	163572.44	344550.96
0.5	431041.3	825673.04
0.6	933006.92	1729334.7
0.7	1774385.36	3263641.8
0.8	3052150.18	5643039.91

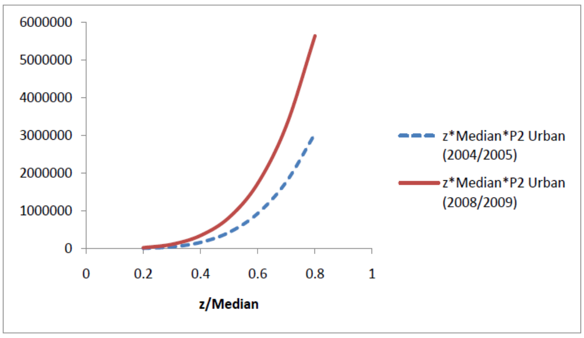

Figure 8: Third order stochastic dominance - Urban 04/05, Urban 08/09.

This figure displays the third order stochastic dominance between urban data collected in 2004/2005 and in 2008/2009. It shows that the urban 2004/2005 stochastically dominated urban 2008/2009 with respect to the severity of poverty and using the relative poverty line, i.e., the severity of poverty for urban 2004/2005 is less than the severity of poverty for urban 2008/2009.

z/Median	Median*P1 Total (2004/2005)	Median*P1 Total (2008/2009)
0.2	7.01	15.6
0.3	29.25	53.8
0.4	72.08	115.51
0.5	150.24	222.86
0.6	273.99	379.77
0.7	445.89	615.23
0.8	664.34	919.06

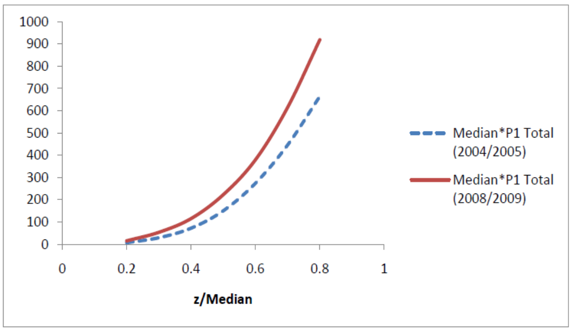

Figure 9: Second order stochastic dominance - Total 04/05, Total 08/09.

This figure displays the second order stochastic dominance between total data collected in 2004/2005 and in 2008/2009. It shows that the total 2004/2005 stochastically dominated total 2008/2009 with respect to the poverty gap ratio and using the relative poverty line, i.e., the poverty gap ratio for total 2004/2005 is less than the poverty gap ratio for total 2008/2009.

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z/Median	z*Median*P2 Total (2004/2005)	z*Median*P2 Total (2008/2009)
0.2	1636.79	14936.41
0.3	26521.34	87730.41
0.4	101696.8	272659.36
0.5	268316.3	639971.9
0.6	579584.36	1297621.35
0.7	1097907.3	2394090.7
0.8	1891982.37	4084855.63

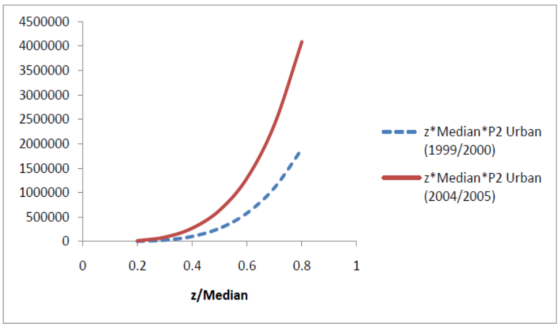

Figure 10: Third order stochastic dominance - Total 04/05, Total 08/09.

This figure displays the third order stochastic dominance between total data collected in 2004/2005 and in 2008/2009. It shows that the total 2004/2005 stochastically dominated total 2008/2009 with respect to the severity of poverty and using the relative poverty line, i.e., the severity of poverty for total 2004/2005 is less than the severity of poverty for total 2008/2009.

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