P versus NP Millennium Prize Problem 2000-2013 Olympics: NP=P for NP-Class by Dedekind-Cut TSP P-Solvability

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Abstract*—The paper presents the P-Solvability characteristics of the TSP, an NP-Complete problem, in a 3D Dedekind-Cut-weight Periodic Table domain as defined in the Zambian PACRA Patent No. 2/2008 . Hence by Cooks theorem, the status of all problems in the NP-Complete Class is NP=P, by virtue of the TSP being a member of the NP-Complete Class having the NP=P solvability status. This African Computer Science finding is opposite to Dr Vinay Deolalikar's 2010 finding of P≠ NP. This settles the 'P versus NP' open millennium prize problem in computer science. The research work was concluded on the 10th Day of August, 2012 before the deadline of 1st January 2013 at 5pm CET USA. The TSP NP=P status is due to the discovery of N_D[dn]= {d1 to dn}, the third missing weighted network dimension. This has for the first time made the crossing of the combinatorial intractability barrier practically possible. N_D[dn], the Dedekind-Cut weight index set is analogous to: the points of a real number line in one-to-one correspondence with the source-node period weights in accordance with the Cantor-Dedekind axiom.

***Index Terms*—Dedekind-cut periodic table, Cantor-Dedekind axiom, millennium prize problem, p vesus np problem, TSP.**

I. INTRODUCTION

The Travelling Salesman Problem (TSP) NP=P status is due to the discovery of **N_D[dn]= {d1 to dn},** the third missing Dedekind-Cut (D-Cut) link-weight network dimension as shown in Figure (a). **N_D** has for the first time made the crossing of the combinatorial intractability barrier practically possible [1]. The P-Solvability constructive proof for the Dedekind-Cut TSP is in six parts: (1) Definitions of Terms; (2) Choice of the TSP, an NP-complete problem, as the prototype for difficult combinatorial optimization problems in the theory of algorithmic complexity; (3) Direct application of the Cook-Levin theorem to the P versus NP Problem status; (4) D-Cut TSP Analysis Problem groupoid formulation; (5) D-Cut TSP Synthesis Problem solution demonstrated using Case 1-n4 in Table I; and (6) Summary of the D-Cut TSP P-Solvability characteristics synthesized from the ten case studies which can be used to design efficient P-Time algorithms for solving the TSP.

A. Millennium Prize Problem Olympics

The P versus NP Millennium Prize Problem Olympics (2000-2013) [53],[54] offer an opportunity for African Computer Science Renaissance. The 'P versus NP' Problem is one of the seven millennium prize problems for which the Clay Mathematics Institute announced the US$1 million prize fund in 2000 if there is a solution to this theoretical computer science problem before 5pm CET USA, Tuesday, January 1, 2013. The seven open problems are: the P versus NP problem; Hodge conjecture; Poincaré conjecture (solved); Riemann hypothesis, Yang-Mills existence and mass gap; Navier-Stokes existence and smoothness; and Birch and Swinnerton-Dyer conjecture.

B. Cantor-Dedekind Axiom Real Number Line

A pictorial representation of a Dedekind-Cut can be achieved by imagining all the positive rational numbers in their natural order marked along a straight line. Dividing this line into two parts such that the part containing the smaller rational numbers contains no greatest provides a picture of a D-Cut [19]. The ND[dn] network link-weight index set is analogous to: the points of a real number line in one-to-one correspondence with the network source-node period weights in accordance with the Cantor-Dedekind axiom[3].

C. Definitions

P stands for polynomial time solvable by computer. NP denotes nondeterministic polynomial-time solvable by computer, that is the computer will continue running forever for large problem sizes requiring combinatorial enumeration. NP-Hard means : Problems to which all members of the NP Class reduce. NP Complete means Problems that are both NP-Hard and members of the NP Class [43],[45] . The family of NP-Hard problems includes virtually all the problems that have frustrated discrete optimizers for decades, such as [43] [45][48]: the directed and undirected Hamiltonian cycle travelling salesman problem[22],[23] , graph-colouring, precedence constrained scheduling, parallel-processor scheduling [17], knapsack, set covering/packing/portioning, various fixed charge models, and 0-1 integer/mixed-integer programming, Dial-a-Ride Problem [5], m-Peripatetic Salesman Problem [10],[20].

NP=P by Cook's Theorem

In 2010 Dr Vinay Deolalikar a mathematician is credited as having solved the computer science "P versus NP" Problem by proving that NP≠P [51][54] . Nevertheless, on 10th August 2012, Dr Lemba Nyirenda an engineer completed his constructive proof using the NP=P finding for the Dedekind-Cut TSP and Cook's theorem. Recalling the Cook-Levin theorem [53], namely: All NP Problems are transformable to the Boolean Satisfiability problem in Logic in P (polynomial) time by a computer. Informally, an NP-complete problem is at least as "tough"as any other problem in the NP Class [53] .

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Therefore if the TSP an NP-Complete Problem is transformed to a P-Solvable Problem (i.e. NP=P for the TSP), then NP=P for all NP Problems. The TSP NP=P constructive proof has taken 24 year to complete (1988 Irvine-California to 2012 Lusaka- Zambia). Reference [29]-[42] represent the individual research discovery trail. The TSP constructive proof is based on concepts drawn from 10 branches of mathematics ([2]-[4],[6]-[8],[14]-[16],[18],[19],[21],[24],[46],[47],[49]-[50]), Alkene chemistry ([11],[25],[27]) , Molecular biology [12] and Operations Research and Management Science ([44], [45]).

D. Solving TSP Under Unsatisfied Hypotheses

The axiom of choice states that any set, including the TSP H∗ set, has a choice function [49]. For example, the number of spanning trees in an n-node complete graph can be as high as nn-2 [7] and the number of directed spanning cycles is (n-1)! ([7],[45]) . Table I shows the TSP and STP Enumerated Spaces for ten cases from [26],[39],[43],[1],[23],[40],[30],[55] and [28] which been studied for the past 24 years (1988-2012). Hence no polynomial-bounded algorithm can be found for the combinatorial TSP and STP problems by enumeration [7],[49] . Thus discrete optimization problems are viewed as problems in combinatorics which are in the "enumeration required category". This led to the P≠NP conjecture [43],[48].
Nevertheless, a conjecture, even a widely held one, is not a proof [43] . It is in this spirit that Hilbert in his 1900 address before the international congress of mathematicians spoke of the conviction: "that every definite mathematical problem must necessarily be susceptible of a precise settlement in the form of an actual answer to the question asked. Sometimes it happens that we seek the solution under unsatisfied hypotheses or in an inappropriate sense and are therefore, unable to reach our goal. Then the task arises of proving the impossibility of solving the problem under the given hypotheses and in the sense required" [6] .

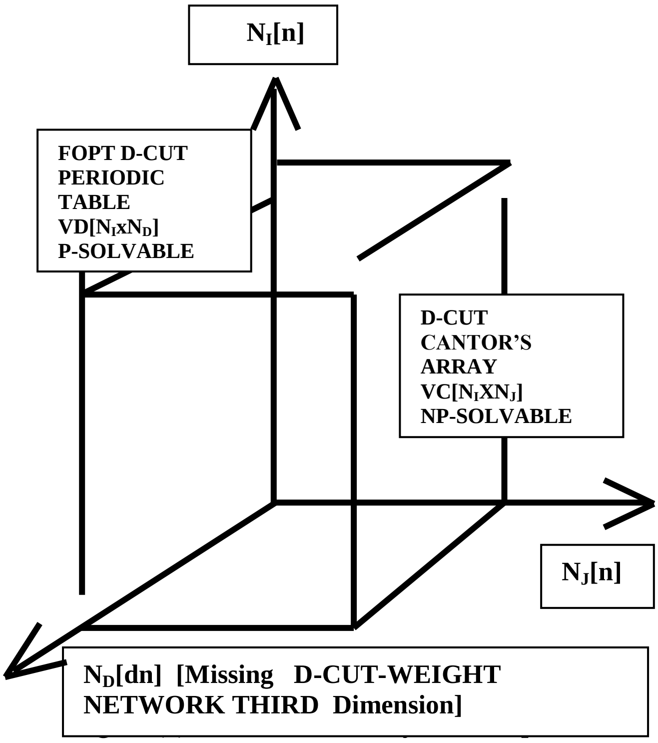
E. Consequences of TSP NP=P

The transformation of the Dedekind-Cut TSP P-solution method into an efficient method for solving NP-complete problems would revolutionize mathematics itself [53],[54] . The TSP NP=P result is of universal interest because of its numerous applications of commercial value in Engineering, Computer Science, Operations Research and Management Science and other allied fields of human endeavour.

II. TSP NP AND P SOLVABILITY

A. Travelling Salesman Problem

The TSP seeks to find the linear span-tree circle (LSC) or tour of connected **n**-links providing a cyclic traversal of total minimum weight in a weighted network of **n** nodes. It is an intractable NP-Complete problem in the 2D weighted network matrix domain, using **(n-1)!** permutated enumerations of the $N_I[n]$ = **{I= 1 to n}** node set to find the $N_I^*[n]$ optimal node ordering providing the optimal TSP solution set (**H*[n], VH*[n])**. The NP-complete permutated node set spaces for network node sizes **n= 4,5,6,8,16,33,**67 are shown in Table I.



B. TSP Solvability 3D Metric Spaces

Let $N_I[n]$ ={ I = 1 to n} (the Source-node index set), $N_J[n]$ ={ J = 1 to n} (the Terminal-node index set), and $N_D[dn]$ = { d_k = d_1 to d_n} (the Dedekind-Cut weight well-ordering index set) be the three dimensions for **VA, VB, VC** and **VD** TSP solvability metric spaces as shown in Fig. (a).
VA[N_I[n] x N_J[n]] is the 2D weighted network matrix. **VB[N_I-Ordered x N_J-Ordered x N_D-Free]** is the 3D Dedekind-Cut weighted component bundle. **VC[N_I-Ordered x N_J-Ordered x N_D-Free]** is the 3D Dedekind-Cut Cantor's Array. **VD[N_D-Ordered x N_I-Ordered x N_J-Free]** is the 3D Dedekind-Cut Periodic Table.

Table I. TSP and STP Enumerated Spaces

Case	NET-WORK NODES: n	TSP LINEAR SPANNING-CIRCLE SPACE : (n-1)!	STP SPANNING-TREE SPACE : n^{n-2}
1	4	6	16
2	5	24	125
3 &4	6	120	1296
5 &6	8	5040	262144
7	16	1.307674368 x 10^{12}	720.58 x 10^{14}
8	17	2.092278989 x 10^{13}	2.862423051 x 10^{18}
9	33	2.631308369 x 10^{35}	1.185583472 x 10^{47}
10	67	5.44344939390 x 10^{92}	495.29 x 10^{116}

Case 1-n4-Asymmetric Network Matrix **(McMillan, 1970)**

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Case 2-n5-Asymmetric Network Matrix **(Nyirenda, 2006)**
Case 3-n6-Asymmetric Network Matrix **(Philips, 1981)**
Case 4-n6-Symmetric Network Matrix **(Barrow,1998)**
Case 5-n8-Asymmetric Network Matrix **(Lawler et al ,1985)**
Case 6-n8-Symmetric Network Matrix **(Philips, 1981; Nyirenda et al, 2008**)
Case 7-n16-Asymmetric Network Matrix **(Nyirenda,1991)**
Case 8-n17-Asymmetric TSP Network Matrix **(http://www.iwr.uni- heidelberg.de/ iwr/ comopt/soft/ TSPLIB95/TSPLIB.html)**
Case 9-n33-Symmetric Network Matrix **(Nemhauser,1988)**
Case 10-n67-Symmetric Network **Matrix (Nemhauser,1988)**

Table II: 2D Nodal Arc-Weight Matrix $VA[N_I[4] \times N_J[4]] = \{W_{IJ}\}$

$N_J[4]$ / $N_I[4]$	J=1	2	3	4	$N_D[d4]$= Æ $FG_I[n]$
I=1	(¥)	-8	**(7*)**	-5	**$FG_1[n]$** =1st PERIOD
2	**(2*)**	(¥)	-6	-4	**$FG_2[n]$** =2rd PERIOD
3	-3	-10	(¥)	**(3*)**	**$FG_3[n]$** =3rd PERIOD
4	-7	**(5*)**	-4	(¥)	**$FG_4[n]$** =4th PERIOD
$FG_I[n]$ ° I th Row/Period/Simplex/Node Functional-Group					

Table III: Case 1-n4 Permutated Node-Set Enumerated Optimal TSP Solution

$N_J[4]$ / $N_I[4]$	**J=1**	**J=2**	**J=3**	**J=4**	**$N_D[4]$ Free $FG_I[n]$**
I=1	(∞) 1-Jan d4	-8 2-Jan d3	**(7*) 3-Jan d2**	-5 4-Jan d1	**$\{W_{1J}\}$ {1/J} $\{d_k\}$free**
I=2	**(2*) 1-Feb d1**	(∞) 2-Feb d4	-6 3-Feb d3	-4 4-Feb d2	**$\{W_{2J}\}$ {2/J} $\{d_k\}$free**
I=3	-3 1-Mar d1	-10 2-Mar d2	(∞) 3-Mar d4	**(3*) 4-Mar d1**	**$\{W_{3J}\}$ {3/J} $\{d_k\}$free**
I=4	-7 1-Apr d3	**(5*) 2-Apr d2**	-4 3-Apr d1	(∞) 4-Apr d4	**$\{W_{4J}\}$ {4/J} $\{d_k\}$free**

Table IV. Case 1-n4 -Cut Nodal Molecular-Weight 3D Cantor's Array $VC[N_I[4] \times N_J[4] \times N_D[d4](free)] = \{(W_{1J})\ I/J\ d_k(free)\}$

Per-muta tion p	**Cyclic Permu tated Node-Set $N_I[n]^P$**	**Figure (b): Hamiltonian-Circle Link-Set $H[n]=\{(I_1,W_{I1/I2}\ d_k, I_2), \cdots,(I_n,W_{In/I1}\ d_k, I_1)\}$**	**VH[n]= H-Value**
1	$\{1,2,3,4\}^1$	{(1,8d3,2), (2,6d3,3), (3,3*d1,4),(4,7d3,1)}	24
2	$\{1,2,4,3\}^2$	{(1,8d3,2), (2,4d2,4), (4,4d1,3), (3,3d1,1)}	19
3	$\{1,3,2,4\}^2$	{(1,7*d2,3),(3,10d2,2), (2,4d2,4), (4,7d3,1)}	28
4*	**$\{1,3,4,2\}^{4*}$**	**{(1,7*d2,3),(3,3*d1,4, (4,5*d2,2),(2,2*d1,1)}**	**17***
5	$\{1,4,2,3\}^5$	{(1,5d1,4), (4,5*d2,2), (2,6d3,3), (3,3d1,1)}	19
6= (n-1)!	$\{1,4,3,2\}^6$	{(1,5d1,4), (4,4d1,3), (3,10d2,2), (2,2d1*,1)}	21
Enumerated Optimal TSP Linear Span Circle (LSC)		**(H*[n],VH*[n]) =$\{(I,W^*_{I/J}\ d_k, J)\}$ LSC = {(2, 2*d1, 1) , (1, 7*d2, 3), (3, 3*d1, 4), (4, 5*d2, 2) } =17***	

$N_I[n=4] = \{1, 2, 3, 4\}$, Source-Nodeset
$N_J[n=4] = \{1, 2, 3, 4\}$, Terminal-Nodeset
N[n=4] = {2, 1, 3, 4}, Optimal TSP Traversal Order*

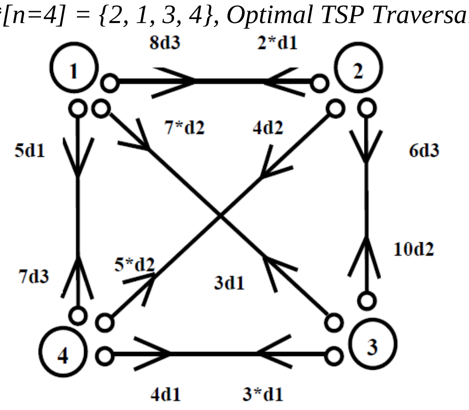

Figure (b). Case 1-n4 Dedekind-Cut-Weight Nodewise-Disconnected Component-Bundle $GD = [N_I[n=4], A(W_{I/J}(dk)), N_J[n=4]]$.

III. D-CUT TSP ANALYSIS PROBLEM

The D-Cut TSP analysis problem involves the determination of the P and NP solvability characteristics of known D-Cut optimal TSP solutions shown in Table I.

A. VC NP-Solvability Metric Space

The D-Cut TSP is NP-Solvable in VA and **VC** because it provides a random distribution of the optimal solution set **(H*[n],VH*[n])** characterized by intractable enumerated spaces as n the nodal network size becomes large.

This is clearly seen in Table I. For example, the enumerated optimal TSP solution for Case 1-n4 is shown in Table III. The component bundle representation of Case 1-n4 is shown in Fig. (b). The Case 1-n4 2D weight matrix representation is shown in Table II and the 3D molecular weight representation is shown in Table IV. Here the permutative random distribution of **(H*[4],VH*[4])** over the four rows can be seen in $VA[N_I[4] \times N_J[4]]$ and $VC[[N_I[4] \times N_J[4] \times N_D[d4]free]$.

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B. VD P-Solvability Metric Space

The D-Cut TSP is P-Solvable in VD because (H*[n],VH*[n]) the optimal solution set linear span cycle (LSC) exhibits four deterministic characteristics that can be formulated as the D-Cut TSP Cumulated Polarization Groupoid Mathematical Programme as shown in Table VII. Here VD is either the fully ordered (FOPT) or partially ordered (POPT). The D-Cut TSP composition factor set can either be the (nx) open-head D1-Path-Factor base set PBX[nx,ny] or the (ny) closed-tail D1-Path-Factor base set PBY[nx,ny] as shown in Table VI. The Case 1-n4 3D periodic table representation is shown in Table V.

Table V. Case 1-n4 D-Cut Fully-Ordered Periodic Table FOPT [DC[4] x FG[4]] = VD[N_I[4] x N_D[d4] x N_J[4](free)]={(W_{1J})I/(J-free) d_k}

DC[4] N_I[4]	DC1	DC2	DC3	DC4	N_J[4] Free FG_I[n]
I=1	-5 4-Jan d1	**(7*)** **3-Jan** **d2**	-8 2-Jan d3	(∞) 1-Jan d4	**{W_{1J}}** **{1/J}** **N_D[4]**
I=2	**(2*)** **1-Feb** **d1**	-4 4-Feb d2	-6 3-Feb d3	(∞) 2-Feb d4	**{W_{2J}}** **{2/J}** **N_D[4]**
I=3	-3 1-Mar d1	**(3*)** **4-Mar** **d1**	-10 2-Mar d2	(∞) 3-Mar d4	**{W_{3J}}** **{3/J}** **N_D[4]**
I=4	-4 3-Apr d1	**(5*)** **2-Apr** **d2**	-7 1-Apr d3	(∞) 4-Apr d4	**{W_{4J}}** **{4/J}** **N_D[4]**
VDC_k	14	**19**	31	**#NAME?**	
L-Bound	**VDC1**	**VDC2**	**VDC3**	**U-Bound**	
D-Cut TSP Optimal-Value (VH*[n]) Bounds					

The submodular algebra mathematical programme consists of : the D1-Path submodular algebra groupoid objective function, subject to: Periodic-Table nodewise (allene-type) / pathwise (diene-type) D1-Path-Factor cumulated-polarization max-overlap ; and three optimality inequalities. These are: the Optimal-value/ D2-Cut-value proximity inequality; the D1-Path-Factor hyperconjugation inequality; and the D-Cut composition series cluster inequality, such that 50-100 percent optimal set elements are located in the (D1+D2)-Cut set composition series independent of the node network size n. Table VII shows the generalized Mathematical Programme and Table VIII shows the programme for Case 1-n4.

Table VI. D-Cut TSP Composition Factor PBX and PBY Base Sets

(1) Fundamental D1-Path Set Composition
Transform the DC1[n] D-Cut set into PFZ[nz,nw] the fundamental D1-Path set.
PFZ[nz,nw] = [DC1[n], (W_{IJ}) I/Jd_1 (max-length-chain)]
= { P_z[nfz], (z = 1 to nz); (nw ≤ nz) }
= (nz)-open-head and (nw)-closed-tail D1-Path set
(2) PBX Open-Head D1-Path Base Set Synthesis
PBX[nx≤nz, ny≤nw] ⊆ PFZ[[nz,nw];
= { P_x[nfx], (x = 1 to nx); (ny ≤ nx) }
= (nx≤nz)-open-head D1-Path set
Such that: Union I_x(PBX) = N_I[n]
P_x[nfx] = A_x + ••• + B_{xy} + C_{xy}
= I_{1x}/J_{1x} + ••• + $I_{(nfx-1)x}/J_{(nfx-1)x}$ + $I_{(nfx)x}/J_{(nfx)x}$
B_{xy} = 1/C_{xy} ; (y= 1 to ny)
(3) PBY Closed-Tail D1-Path-Factor Base Set Synthesis
PBY[nx≤nz, ny≤nw] = { P_y[nfy], (y = 1 to ny); (ny ≤ nx)}
= (ny≤nw)-closed-tail D1-Path set
P_y[nfy]= [{ P_x[nfx]} ⊆ PBX, B_{xy}-C_{xy} Common (÷)]

Table VII. D-Cut TSP Cumulated Polarization Groupoid Mathematical Programme

Groupoid Objective Function:
(H*[n],VH*[n]) = [FOPT*(PBX[nx , ny]), LSC(÷)]
or [FOPT*(PBY[nx , ny]), LSC(÷)]
LSC(÷) = Linear Span-tree Circle submodular division
Subject to Optimality Conditions:
(1) PBX /PBY FOPT Cumulated- Polarization max-Overlap:
(a) Nodewise (allene) =[{I_x}$^{\underline{nI}}$/{(J $_{EQUAL}$ d_k)}$^{\underline{nJ}}$;
(d_k Gradient :S2 to S1)] * for (ny ≤ 4).
(b) Pathwise (diene) =[{I_{y1}}^{1I}/{$J_{NOT-EQUAL}$ d_k}$^{\underline{xJ}}$; (d_k Gradient: S1to S2)] * ; **(I_{y1} ∈ P_{y1}) and (**J∈**P_{y2}**) for (ny > 4).
(2) Optimal-Value D2-Dedekind-Cut Value
Proximity Inequality
(VDC_1[n]) <(VDC_2[n] = [VH* [n] ± Δ]) <(VDC_3[n]) (VDC_1[n]) < (VH* [n]) < (VDC_3[n])
(3) D1-Path-Factor Hyperconjugation Inequality
(ny) ≤ nD1-Path-Factor(+)* ≤ (nx + 1)
(4) k^{th} Order Dedekind-Cut Composition Series Cluster Inequality
For k = 1 to n :
ng_k* / n = nd_1*/n + nd_2*/n + ••• + nd_k*/n
0/1 < ng_1* / n ≤ n/n
1/2 **= (50%) < ng_2*/ n ≤ n/n (100%) ≡ [Min nd_{2+}]**
2/3 < ng_3* / n ≤ n /n
••• ••• •••
(k-1)/k < ng_k* / n ≤ n/n

Table VIII. Case 1-n4 D-Cut TSP Cumulated Polarization Groupoid Mathematical Programme

Linear Span Tree (LST):
T* [4] = [(2*) $\underline{2}^{2J2d2}$/1d1 + (5) 1/$\underline{4}$ d1]
÷ [(3*) 3/(1,$\underline{4}$)d1 + (4) 4^{2J2d1} /3d1]
Linear Span Circle (LSC):
H* [4] = [(2*) 2^{2J2d2}/1d1+ (7*) 1/3d2]
÷ [(3*) 3/4 d1 + (5*) 4^{2J2d1}/2d2]
VH* [4]=2* + 7* + 3* + 5*=17*
Subject to Optimality Conditions:
PBX Nodewise (allene-type) Cumulated-Polarization max-Overlap: [{I_x}$^{\underline{nI}}$/{($J_{EQUAL}$$d_k$)}$^{\underline{nJ}}$; ($d_k$:S2,S1)] *

= [{1,$\underline{3}$}$^{\underline{2I}}$/{ $\underline{4}$d1}$^{\underline{2J}}$(d1:S1)] *;[{3,$\underline{4}$}$^{\underline{2I}}$/{ $\underline{2}$d2}$^{\underline{2J}}$; (d2:S1)]*
Optimal-Value D2-Dedekind-Cut-Value Proximity Inequality
(VDC_1[n] =14) <(VDC_2[n] = 19) < (VDC_3[n]= 31)
(VDC_2[n] = 19) = [VH* [4] ± Δ]) = [17* + 2])
D1-Path-Factor Hypercongugation Inequality
(ny) ≤ nD1-Path-Factor(+)* ≤ (nx+ 1)
(1) < nD1-Path-Factor(+)* = 2 = (2)
kth Order Dedekind-Cut Composition Series
Cluster Inequality
(k-1)/k < ng_k***** / n ≤ n/n
0/1 < ng_1***** / n = [(ng_0*****=0)/4 + (nd_1*****=2)/4] = 2/4 < 4/4
1/2 < ng_2*/ n =[(ng_1*=2)/4 + (nd_2*=2)/4] = 4/4

IV. D-CUT TSP SYNTHESIS PROBLEM

The D-Cut TSP synthesis problem involves the application of the P- Solvability characteristics of known D-Cut TSP optimal solutions to design simple and efficient TSP solution methodologies for any D-Cut weight network.

A. Min-nd2+ Groupoid TSP Solution Method

The TSP optimal solution (H*[n],VH*[n]) in the Dedekind-Cut weight Periodic Table (FOPT or POPT) domain is determined as follows.

Case	n	[nD1-F ÷ nx] ratio	(ny)≤ nD1-Hyper-Conj-Factor* ≤ (nx+ 1) *[nD1-Hyper-Conj-Factor* around nx]*
1	**4**	**2/1**	(1) < **nD1-Hyper-Conj-Factor*** = 2 = (2)
2	**5**	**2/2**	(2) < **nD1-Hyper-Conj-Factor*** = 2 < (3)
3	**6**	**2/3**	(1) < **nD1-Hyper-Conj-Factor*** = 2 < (4)
4	**6**	**2/3**	(2) < **nD1-Hyper-Conj-Factor*** = 2 < (4)
5	**8**	**2/2**	**(1)** < **nD1-Hyper-Conj-Factor* = 2 < (3)**
6	**8**	**3/3**	(2) < **nD1-Hyper-Conj-Factor*** = 3 < (4)
7	**16**	**5/6**	(2) < **nD1-Hyper-Conj-Factor*** = 5 < (7)
8	**17**	**6/6**	(2) = **nD1-Hyper-Conj-Factor*** = 6 < (7)
9	**33**	**9/15**	**(8)** < **nD1-Hyper-Conj-Factor* = 9 < (16)**
10	**67**	**19/26**	**(21) < nD1-Hyper-Conj-Factor* = 19 < (27)**

(H*[n],VH*[n]) = min-value { (H_m[n],VH_m[n]) ;
m=0 to m_{max} ≤ (nx +2)}

where

(1) m=0

T_0[n]=[FOPT(Natural-Order), DC1 LST(÷)]

(H_0[n],VH_0[n]) = [T_0[n], LSC(÷)]

(2) m=1

T_1[n]=[FOPT(PBX[nx , ny]), max nd_1 LST(÷)]

(H_1[n],VH_1[n]) = [T_1[n], LSC(÷)]

(3) m=2 to m_{max} ≤ (nx+2)

T_m[n]=[FOPT(PBX[nx , ny]), Cum-max-overlap LST(÷)]

(H_m[n],VH_m[n]) = [T_m[n], min nd_{2+} LSC(÷)]

B. Case 1-n4 Complete TSP Groupoid Solution

The complete TSP groupoid solution for Case 1-n4 follows. The reference parameters are obtained from FOPT shown in Table V.

FOPT(T_0) in {Natural} N_I[n] - order
T_0 [4]=(5)1/4 d1 ÷(2) 2/1d1 ÷ [(3) 3/(1,4)d1+ (4)4 /3 d1]
H_0[4]=(8)1/2 d3 ÷(6) 2/3d3 ÷[(3)3/4*d1 + (7) 4/1 d3]
VH_0 [4]=8 + 6 + 3 + 7 = 24
nd_{2+}/n = 3/4

FOPT (T_1) in { max nd_1 } N_I[n]-order
T_1 [4]=(2)2/1d1 + (5)1/4 d1 + (4)4 /3 d1 + (3) 3/(1,4)d1
H_1[4]=(2)2/1*d1 + (5)1/4 d1 + (4)4 /3 d1 + (10) 3/2 d2
VH_1 [4]=2 + 5 + 4 + 10 = 21
nd_{2+}/n = 0/4

FOPT(T_2) in {CumD1[2/$\underline{1}$d1,3/$\underline{1}$d1:$\underline{1}^{2d1}$/4d1]} N_I[n]–Order
T_2[4]=[(2)2/$\underline{1}$d1] ÷[(3)3/($\underline{1}$,4)d1 + (5)$\underline{1}^{2d1}$/4 d1 + (4)4 /3d1]
H_2 [4]=[(6)2/3d3] ÷ [(3)3/1 d1 + (5)1^{2d1}/4 d1 + (5)4/2d2]
VH_2 [4]=6 + 3 + 5 + 5 = 19
Min: nd_{2+}/n = ¼

FOPT(T_3) in {CumD1[1/$\underline{4}$d1,3/$\underline{4}$d1:$\underline{4}^{2d1}$/3d1] and CumD2[3/$\underline{2}$d2,4/$\underline{2}$d2:$\underline{2}^{2d2}$/1d1]} N_I[n]– Order
T_3[4]=[(2*)$\underline{2}^{2d2}$/1d1+(5)1/$\underline{4}$d1]÷[(3*)3/($\underline{4}$)d1+(4) 4^{2d1}/3d1]
H_3[4]=[(2*)2^{2d2}/1d1+(7*)1/3d2]÷[(3*)3/4 d1+(5*)4^{2d1}/2d2]
VH_3 [4] = 2* + 7* + 3* + 5*= 17*
Min: nd_{2+}/n = 0/4

(H*[4],VH*[4]) = min-value { (H_m[4],VH_m[4]) ;
m= 0 to m_{max} = 3}
= (H_3[4],VH_3[4])

V. CONCLUSION

The TSP is P-Solvable in the fully-ordered (FOPT) or partially ordered (POPT) Dedekind-Cut weight periodic table. The D-Cut TSP P-Solvability characteristics for the ten cases are summarized in Table IX, X and XI below. These characteristics represent the TSP Analysis Problem and the TSP Synthesis Problem results.

A. D1-Path-Factor Hyperconjugation Inequality

Table IX. Distribution of the D1-Path-Factor Hyperconjugation for the ten cases

B. kth Order D-Cut Composition Series Cluster Inequality

Table X. Distribution of Dedekind-Cut Composition Series Clusters versus Enumerated TSP spaces for the ten case

Case No. & Cluster Ratio: ng_2*/ n	n	FOPT Optimal Solution Composition Series Clusters : (k-1)/k ≤ ng_k* / n ≤ n/n	Maximum FOPT D-Cut Space Cluster Ratio: $d_k$max*/dn	Enumerated TSP Space: (n-1)!
1 **100%**	4	**0/1 < ng_1* / 4 = 2/4 < 4/4** **1/2 < ng_2* / 4 = 4 /4 (100%)**	**d2*/d4 =50%**	**6**
2 **80%**	5	**0/1 < ng_1*/ 5 = 2/5 < 5/5** **1/2 < ng_2*/5 = 4/5 (80%)< 5/5** **2/3 < ng_3* / 5 = 5/5**	**d3*/d5 =60%**	**24**
3 **83%**	6	**0/1 < ng_1* / 6 = 3/6 < 6/6** **1/2 < ng_2* /6= 5/6(83%) < 6/6** **2/3 < ng_3* / 6= 5/6 < 6/6** **3/4 < ng_4* / 6= 6/6**	**d4*/d6 =67%**	**120**

4 **83%**	6	$0/1 < ng_1^*/6 = 3/6 < 6/6$ $1/2 < ng_2^*/6 = 5/6(83\%) < 6/6$ $2/3 < ng_3^*/6 = 6/6$	**d3*/d6** **=50%**	**120**
5 **100%**	8	$0/1 < ng_1^*/8 = 3/8 < 8/8$ $1/2 < ng_2^*/8 = 8/8\ (100\%)$	**d2*/d8** **=25%**	**5040**
6 **100%**	8	$0/1 < ng_1^*/8 = 5/8 < 8/8$ $1/2 < ng_2^*/8 = 8/8\ (100\%)$	**d2*/d8** **=25%**	**5040**
7 **88%**	16	$0/1 < ng_1^*/16= 9/16 < 16/16$ $1/2 < ng_2^*/16=14/16(88\%) < 16/16$ $2/3 < ng_3^*/16= 14/16 < 16/16$ $3/4 < ng_4^*/16 = 16/16$	**d4*/d16** **=25%**	**1.3076** **74368** **$\times 10^{12}$**
8 **82%**	17	$0/1 < ng_1^*/17=11/17 < 17/17$ $1/2 < ng_2^*/17=14/17(82\%) < 17/17$ $2/3 < ng_3^*/17= 16/17 < 17/17$ $3/4 < ng_4^*/17= 17/17$	**d4/d17** **=24%**	**2.0922** **78989** **$\times 10^{13}$**
9 **88%**	33	$0/1 < ng_1^*/33= 19/33 < 33/33$ $1/2 < ng_2^*/33=29/33(88\%) < 33/33$ $2/3 < ng_3^*/33= 31/33 < 33/33$ $3/4 < ng_4^*/33= 32/33 < 33/33$ $4/5 < ng_5^*/33= 33/33$	**d5*/d33** **=15%**	**2.6313** **08369** **$\times 10^{35}$**
10 **93%**	67	$0/1 < ng_1^*/67= 38/67 < 67/67$ $1/2 < ng_2^*/67=53/67(93\%) < 67/67$ $2/3 < ng_3^*/67= 64/67 < 67/67$ $3/4 < ng_4^*/67= 67/67$	**d4*/d67** **=6%**	**5.443449** **39390** **$\times 10^{92}$**
n ranging from 4 to 67 nodes		*ng_2^*/n: ranges from 80% to 100%*	*$d_k max$ *ranges from d2 to d5*	

C. D2-Dedekind-Cut-Value Optimal-Value Proximity Inequality

Table XI. istribution of the D2-Dedekind-Cut-Value Optimal-Value Proximity Locations for the ten cases

Case	n	VDC1	[VH*[n] ÷ ςΔX2] ratio	VDC2 = [VH*[n] ± Δ]	VDC3
1	4	14	1.529	19= [17* + 2]	31
2	5	6	1.091	12 = [11* + 1]	19
3	6	43	1.222	77= [63* + 14]	127
4	6	21	1.036	29= [28* + 1]	46
5	8	9	1.25	20= [16* + 4]	36
6	8	2320	1.075	3300= [3070* +230]	3960
7	16	93	1.318	170 = [129* + 41]	285
8	17	0	1.923	75=[39* +36]	120
9	33	8613	0.979	10745 = [10971 * – 226]	13353
10	67	1257	0.985	1590 = [1615* –25]	2008
VH[n] around VDC2 between VDC1 and VDC3 bounds*					

Here we see independent of **n** the node network size: (1) VH*[n] is around VDC2 between VDC1 and VDC3 bounds; (2) the ng_2^*/n D-Cut cluster ratio ranges from 80% to 100% ; (3) the number of hyperconjugate D1-Path-Factors* is around (nx) acyclic -heads between (ny) cyclic-tails and (nx+1) ;(4) the maximum D-Cut index ($d_k max$ *) ranges from (d2*) to (d5*) suggesting a fit-for-purpose partially ordered Dedekind-Cut periodic table (POPT); and (5) the hyperconjugate D1-Path* Composition Factor [2] base sets [24] exhibit allene- [11][27] or diene-type [11][27] min-nd_{2+} cumulated polarization max-overlap in FOPT* or POPT*.

By the axiom of choice [49] these P-Solvability characteristics allow the design of many different P-Time algorithms for the D-Cut TSP Synthesis problem. Thus the TSP solvability status is NP = P. Hence by Cook's theorem: The solvability status of the whole NP-Class is NP=P. This 2012 finding settles the P versus NP Millennium Prize Problem Olympics from 2000 to 2013. This historical finding in computer science is mainly due to the discovery of the missing D-Cut weight network dimension, namely : $\mathbf{N_D}$ ={ d1 to dn}, the Cantor-Dedekind axiom [3] real number line. $\mathbf{N_D}$ has for the first time made the crossing of the combinatorial intractability barrier practically possible[1].

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