Towards an Accurate Definition of the Local Geoid Model in Egypt using GPS/Leveling Data: A Case Study at Rosetta Zone

Essam M. Al-Krargy, Mohamed I. Doma, Gomaa M. Dawod

Abstract*— Nowadays the Global Positioning System (GPS) is one of the most favorite techniques in practical geodesy. A major dilemma in GPS surveying lies in its ellipsoidal-based type of heights, while in engineering practice orthometric heights are usually utilized. Thus, it is important to convert GPS heights into orthometric heights through applying an accurate geoid model. The objectives of this paper are to model a local geoid in the study area using GPS/levelling technique, and to evaluate the performance of several Global Geopotential Models (GGMs) particularly the OUS-91A, EGM96 and EGM2008 in the study area, which is located in the northern Egypt at Rosetta zone area. The accomplished results show that the EGM 2008 represents the most precise global geopotential model to be used for geoid determination in Egypt. Furthermore, the achievable accuracy of local geoid determination in the study area after using regression method models is ranges between 0.059 meter to -0.083 meter, with an average -0.01 meter and standard deviation of ± 0.05 meter. It is concluded that increasing the number of control points with well spatial distribution will result in developing a precise geoid model for Egypt.

***Index Terms*— Global Geopotential Models (GGMs), Global Positioning System (GPS), Local geoid. Orthometric height.**

I. INTRODUCTION

During the last decades, GPS has been used in many applications of geodesy, geophysics, and surveying. GPS technique provides surveyors with three-dimensional coordinates including the height relative to a best fitting ellipsoid that is called the ellipsoidal height (h). On the other hand, many applications in civil engineering, such as topographic mapping, engineering design, and construction projects, depend upon the so-called orthometric height (H), which is the height above the geoid as approximated by the Mean Sea Level (M.S.L). The geoid fits the actual Earth surface more accurately than the geometric representation of the earth by the ellipsoid. Because of these facts ellipsoidal heights can't satisfy the needs of practical surveying engineering or geophysical application as they have no physical meaning and must be transformed to orthometric height. In order to accomplish this height transformation, there is a need to calculate the difference between ellipsoidal and orthometric heights, which is called the geoid undulation N.

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These undulations can be determined using several techniques such as: Global Geopotential Models (GGMs) such as the OSU-91A, EGM 96, EGM 2008 models, gravimetric method using surface gravity data, and geometric method using GPS/leveling data (Erol and Çelik, 2004). The present research aims to develop a precise local geoid using GPS/levelling data, and to compare the attained geoid undulations against some GGMs in order to evaluate their precision performance in the study area. The paper is organized as follows; first, an overview of geoid modeling is presented. Then, the most used GGMs in Egypt are summarized. Thirdly, a numerical case study is outlined and the accomplished results are discussed. Finally, conclusions and recommendations are given.

II. GEOID MODELING

The geoid is the equipotential surface of the Earth's gravity field approximating Mean Sea Level (MSL) in an optimum way, and extended under the continents. The geoid is determined using several techniques based on a wide variety of using one or more of the different data sources such as: the gravimetric method using surface gravity data, satellite positioning based on measuring both ellipsoidal heights for stations with known orthometric heights (Al-Ghamdi and Dawod, 2013):

$$N = h - H \tag{1}$$

Where N is the geoidal heights or geoidal undulations, h is the ellipsoidal or GPS-based height, and H is the orthometric or MSL-based height. Fig. (1) shows the relationship between ellipsoidal, orthometric and geoid heights.

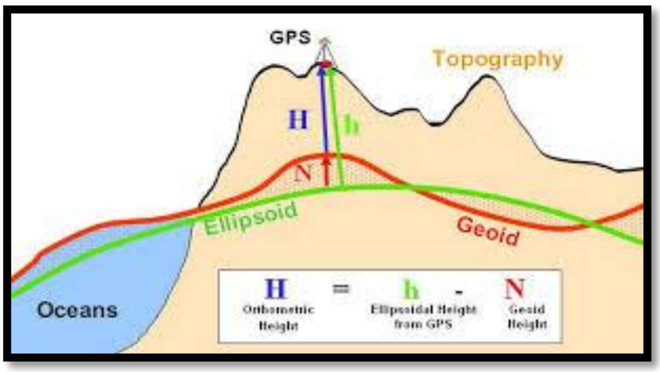

Fig. 1: The Relation between Ellipsoid Height and Geoid Height

According to the geometric method, there are several factors that affect the accuracy of the determined geoid model, such as (Kaloop and Rabah, 2008).

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- Distribution and number of reference stations (GPS/leveling stations). These points must be distributed homogeneously to the coverage area of the model and have to be chosen to Fig. out the changes of geoid surface.
- The accuracy of GPS derived ellipsoidal heights (h) and the heights derived from leveling measurements (H).
- Used method while modeling the geoid.

Several techniques are used for determination of geoid surface surround the data points, for example, interpolation, finite elements, numerical differential solution, one-dimensional datum transformation. Interpolation methods are the most common approaches utilized for modeling the geoid heights (N) for a local area. There are different interpolation algorithms, such as (Maher et al, 2012):

- Inverse Distance Weighting to Power
- Kriging
- Minimum Curvature
- Nearest Neighbor
- Polynomial regression

In this study, the last method (polynomial regression) has been chosen to model the geoid undulations in the study area. This method is described as:

$$N(\varphi,\lambda) = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\phi_i\lambda_j \qquad (2)$$

Where a_{ij} are the coefficients of the polynomial regression, n, m are the polynomial orders, φ is the geocentric latitude, and λ is the geocentric longitude. This method is used to define large-scale trends and patterns in data, and there are several options to define the type of the trend surface. Such options are presented in equations 3, 4, 5, and 6.

Simple Planar Surface:

$$N = a_o + a_1\varphi + a_2\lambda \qquad (3)$$

Bi-Linear Saddle:

$$N = a_o + a_1\varphi + a_2\lambda + a_3\varphi\lambda \qquad (4)$$

Quadratic Surface:

$$N = a_o + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\lambda^2 + a_5\varphi\lambda \qquad (5)$$

Cubic Surface:

$$N = a_o + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\lambda^2 + a_5\varphi\lambda + a_6\varphi^3 + a^7\lambda^3 + a_8\varphi^2\lambda + a_9\varphi\lambda^2 \qquad (6)$$

Where $a_0, a_1, a_2, \ldots\ a_n$ *are the* coefficients of polynomial regression, φ is the geocentric latitude, λ is the geocentric longitude. In practice, for surfacing with polynomial regression, selection of polynomial degree is very important. The surface can be lost its reality and suitability due to wrong selection of coefficient and polynomial degree. In surfacing with polynomial regression methods, degree of polynomial is depending on number of points and degree of freedom. As possible as it must be started with the highest degree and the most suitable coefficient must be determined by using statistical tests (Schut, 1976, Yanalak, 1991).

III. GLOBAL GEOPOTENTIAL MODELS (GGMs)

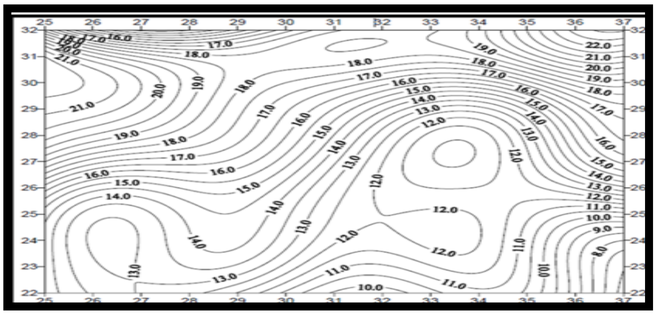
During the last 30 years, numerous GGMs have been computed by various groups. For an extensive description of existing model refer to the web pages of international center for Global Earth Models (ICGEM 2014), and the references therein. Examples of the most common GGMs applied for geoid modeling computations are The Earth Geopotential Models (EGM 2008), (EGM 96), and the OhioStateUniversity (OSU-91A). The quality of the selected GGMs used greatly affects the accuracy of the computed geoid. These global models have been developed from the combination of satellite perturbation analysis with both surface gravity and satellite altimetry data. A brief description for three GGMs the most common worldwide, and also in Egypt, is follows:

A) Ohio State University (OSU-91A) Model

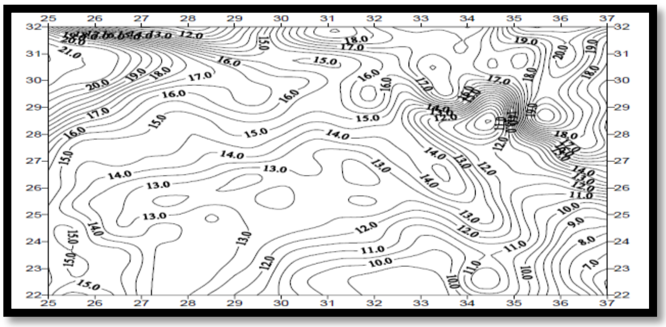
To cater for the needs of the GPS system, the department of geodetic sciences and surveying of the Ohio state university established the model OSU-91A, from the model GEM-T2 and from available gravity data (for more detailed of OSU-91A see Rapp et al., 1991).The OSU-91A geopotential model was for long time the most accurate reference model in geodesy and other geosciences. This model provides the spherical harmonics up to degree and order 360. Therefore, the shortest wavelength of this model is one degree in latitude and longitude, and its resolution is one-half degree (about 50 km). OSU-91A did not incorporate enough data from the Middle East, and consequently it may not be able to represent it very well. The geoid undulations of OSU-91A over Egypt area are represented as a contour map in Fig. (2a) (Saad and Dawod, 2002).

B) Earth Geopotential Model 1996 (EGM 96)

The Earth Geopotential Model 1996 (EGM 96) is a GGM produced through a collaborative effort of the NASA Goddard Space Flight Centre (GSFC), the National Imagery and Mapping Agency (NIMA), and the Ohio State University (OSU). EGM 96 was improved the data holding over many of the world's land area. Furthermore, it has been calculated using a new set of 30′ × 30′ mean free-air gravity anomalies obtained from several ocean gravimetric missions which took place from 1975 to 1990 (Catlao and Sevilla, 1999). In addition, EGM96 incorporated improved surface gravity data, altimeter-derived anomalies from ERS-1 and from the GEOSAT Geodetic Mission, extensive satellite tracking data, the GPS NASA’s Tracking and Data Relay Satellite System (TDRSS), the French DORIS system, the US Navy TRANSET Doppler tracking system, as well as direct altimeter ranges from TOPEX/POSEIDON (T/P), ERS-1, and GEOSAT. The distribution and extent of the surface gravity data in EGM 96 is a major improvement on the data available for the OSU-91A. The geoid undulations of EGM 96 GGM over Egypt are represented as a contour map in Fig. (2b) (Saad and Dawod, 2002).



(a)



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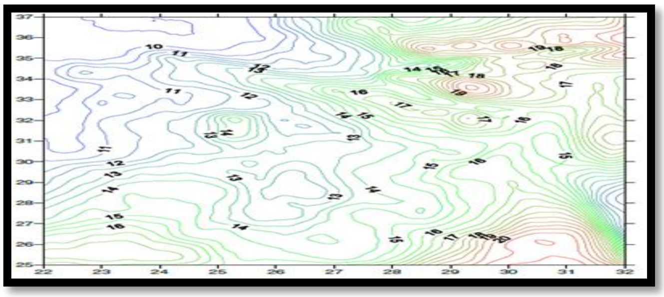

(c)

Fig. 2: (A) A Contour Map of Geoid Undulations of OSU9-1A (Saad and Dawod, 2002), (B) A Contour Map of Geoid Undulations of EGM 96 (Saad and Dawod, 2002), and (C) A Contour Map of Geoid Undulations of EGM2008 in Egypt (Rabah and Kaloop, 2013)

C) Earth Geopotential Model 2008 (EGM 2008)

EGM 2008 is the most recent earth geopotential model developed by least squares combination of the ITG-GRACEEO35 gravitational model and its associated error covariance matrix, with the gravitational information obtained from a global set of area –mean free-air gravity anomalies defined on a 5-arc-minute grid. This grid was formed by merging terrestrial, altimetry-derived, and airborne gravity data. Over areas where available, their spectral content was supplemented with gravitational information implied by the topography (Pavlis et al, 2008; Avlis et al, 20120) It does not incorporate any GPS/Leveling or Astronomic deflections of the vertical data (Dawod et al, 2010). For the first time, this gravitational model is complete to spherical harmonic degree and order 2159, and contains additional coefficients extending to degree 2190 and order 2159. The spatial resolution (half wavelength) of EGM2008 is (nominally) 9.3x9.3 km on the equator, which is 6 times higher than that of EGM996. Over EGM96 and older GGMs, EGM 2008 represents improvement by a factor of six in resolution, and by factors of three to six in accuracy depending on gravitational quantity and geographic area. EGM2008 represents a milestone and a new paradigm in global gravity field modeling, by demonstrating for the first time ever, that given accurate and detailed gravimetric data, a single global model may satisfy the requirements of a very wide range of applications (Pavlis et a;; 2012). The geoid undulations of Egypt as computed from EGM 2008 are shown in Fig. (2c) (Rabah and Kaloop, 2013).

IV. STUDY AREA AND AVAILABLE DATA

The study area is located in the northern Egypt from Rosetta to Brullous Fishing port along the Mediterranean coast (Fig. 3).It extends from longitude 30º22.40' E to 30º55.15' E, and from latitude 31º 27.8' N to 31º 33.9' N. The current area of the study area covers 198 square kilometers approximately. The study have been carried out using 31 GPS/leveling data points as shown in Fig. (4). 24 of these points were used as control points in the development of a geoid model for the Rosetta area, while 7 points were used as check points. The selection of control points set was based upon maintaining homogeneous distribution taking into consideration the topography of the region. The majority of this data set comes from research project carried out by the Survey Research Institute (SRI). The spirit levelling observations were performed as closed loops that run between known high-precision benchmarks established by Egyptian survey Authority (ESA) based on the national vertical datum of Egypt, whose origin is based on Mean Sea Level (MSL) at Alexandria tide gauge of 1906. In addition, GPS measurements were carried out relative to the ESA national geodetic reference framework of Egypt

Fig. 3: The Study Area

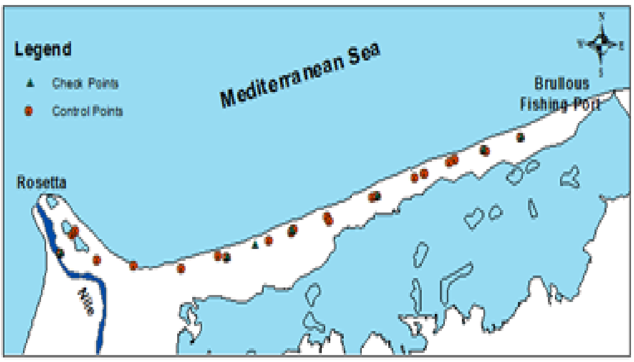

Fig. 4: Distribution of Control and Check Points in the Study Area

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V. EVALUATION OF GGMS IN THE STUDY AREA

In order to evaluate the performance of selected GGMs in the study area: first geoid undulations at all GPS/leveling points (Nobs) have been computed using equation 1, secondly the Trimble Business Center program (TBC) was used to calculate geoid undulations (NGGMs) at the same points for OSU-91A, E9M96, and EGM2008, and thirdly a comparison of results has been performed in order to choose the optimum GGM to be utilized in Rosetta zone.

$$dN = N_{obs} - N_{GGMs} \qquad (7)$$

where:

N_{ObS} is the difference between ellipsoidal heights "h" and orthometric heights "H"

N_{GGMs} is geoid undulations for Geoid Global Models (OSU-91A, EGM 96, EGM 2008)

The obtained results are presented in Tables (I) and (II). Fig. (5) shows the geoid undulations (NGGMs) for the OSU-91A, EGM96, and EGM2008 and (Nobs.) from GPS/Levelling observation. Fig. (6) shows the geoid undulation differences (dN) between (Nobs) and (NGGMs). It can be noticed that, the EGM 2008 is the closer GGMs to the observed local geodetic dataset, in terms of difference of geoid undulation that have a maximum of 0.691 meter, a minimum of 0.430 meter, an average of 0.571 meter, and a standard deviation of ±0.060.

Table I: Difference of Geoid Undulation (dN) between GPS/Leveling and GGMs

St.	dN			St.	dN		
	OSU-91A	EGM 96	EGM 2008		OSU-91A	EGM 96	EGM 2008
R1	1.715	0.714	0.577	R14	2.182	0.885	0.652
R2	1.843	0.797	0.651	R15	2.117	0.816	0.582
R3	1.710	0.667	0.523	R16	2.160	0.830	0.586
Ch 1	1.746	0.701	0.554	Ch 4	2.076	0.776	0.542
R4	1.808	0.728	0.575	R17	2.262	0.879	0.624
R5	1.744	0.656	0.503	R18	2.302	0.912	0.650
R6	1.865	0.744	0.583	Ch 5	2.242	0.895	0.647
R7	1.818	0.697	0.538	R19	2.269	0.839	0.572
Ch 2	1.991	0.805	0.615	R20	2.242	0.823	0.536
R8	1.910	0.776	0.612	R21	2.262	0.778	0.481
R9	1.867	0.681	0.490	Ch 6	2.259	0.876	0.620
R10	1.966	0.780	0.590	R22	2.334	0.848	0.544
R11	1.953	0.762	0.570	R23	2.218	0.736	0.430
R12	2.100	0.846	0.630	Ch 7	2.330	0.947	0.691
Ch 3	2.056	0.755	0.524	R24	2.274	0.774	0.467

Table II: Statistical Function for Geoid Undulations Difference (dN) Computed usingOSU-91A, EGM96, and EGM2008

Statistical function	dN		
	OSUA91	EGM 96	EGM 2008
Minimum	1.710	0.656	0.430
Maximum	2.334	0.947	0.691
Range	0.624	0.291	0.261
Average	2.051	0.789	0.571
St. Deviation	0.208	0.074	0.060

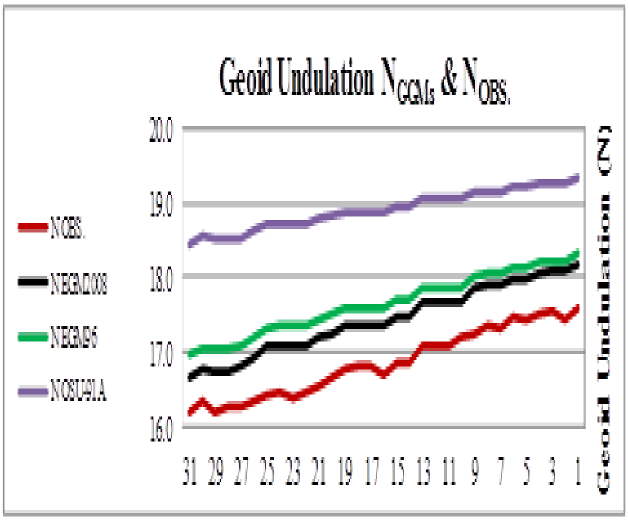

Fig. 5: Result of the Comparison of (NEGM2008), (NEGM96), (NOSU-91A), and (NObS)

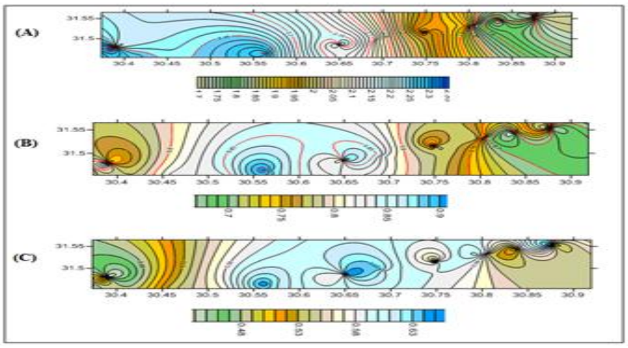

Fig. 6: (A) A Contour Map of dN between (NOSU-91A, NObs.), (B) A Contour Map of dN between (NEGM96, NObs.), and(C) (Contour Map of dN between (NEGM2008, NObs.)

VI. DEVELOPMENT OF A LOCAL GPS/LEVELLING GEOID MODEL

The second step of the processing stage of the current study is to develop the local geoid model over the study area by using GPS/leveling observations. Many surface modeling techniques can be used to improve the accuracy of the model. In this study, the polynomial regression approach is utilized since is the most common method for local geoid surface modeling. The computational steps are outlined as follows Fig. (7):

-Four cases are processed by using 6, 12, 18 and 24 data points respectively in order to investigate the effect of point distribution.

-Additionally, three types of polynomial order (1, 2, and 3) have been performed in order to investigate the optimum polynomial order in geoid modeling. Of course, in the first case (6 control points) the 2rd, and 3rd order polynomial regression have not been applied because the number of unknowns is more than the number of equations.

-Thus, 10 local geoid models have been obtained as a combination of all possible cases.

-Finally, a comparison has been carried out between the values of geoid undulations of each local model (NLocal_Model), and geoid undulations of the GPS/Levelling (NObs) at the check points.

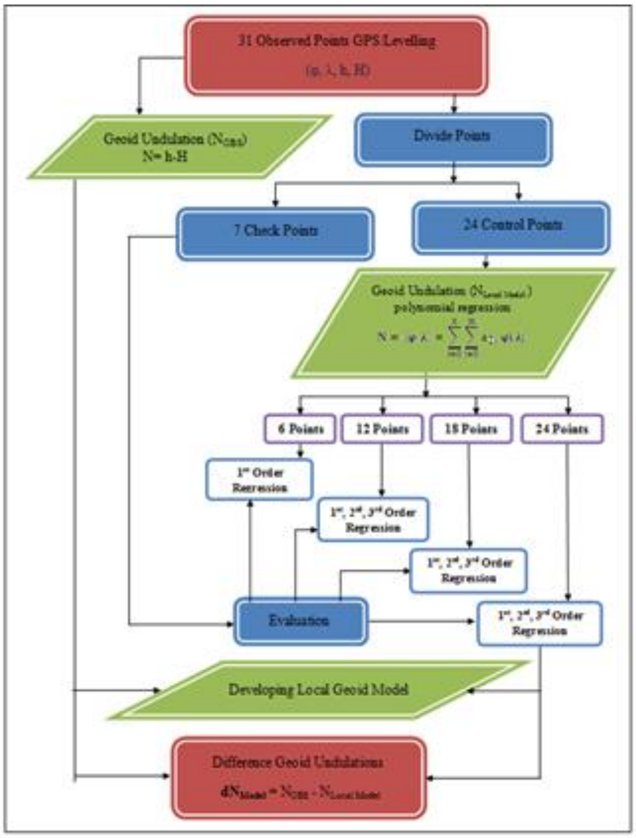

Fig. 7: A flowchart of the Utilized Methodology

Fig. (8), and (9) along with Table (III) depict the accomplished results.

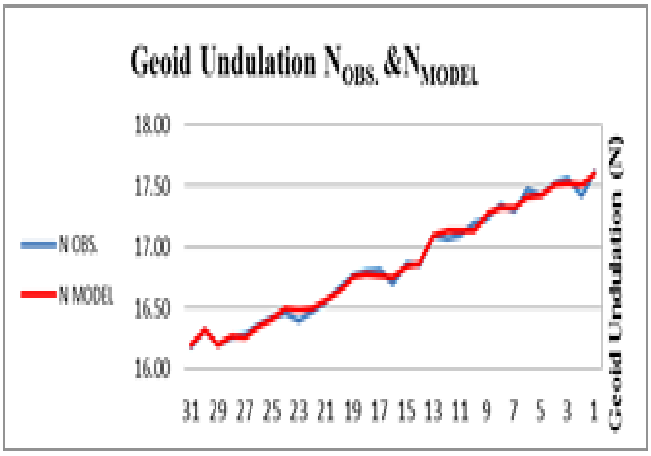

Fig. 8: Result of the Comparison of (NOBS) and (NLocal -Model)

The accomplished results show that the best model for local geoid in the study area is that of the 2rd-order polynomial degree in the fourth case (24 control points).

The attained geoid undulation of that model (NLocal_Model) is computed by:

$$N_{local\_model} = -73856.5 + 5661.6\phi - 1008.1\lambda + 42.33\phi\lambda + 0.6668h - 110.32\phi^2 - 5.299\lambda^2 - 0.016h^2 \qquad (8)$$

Table III: Statistical Parameters of Geoid Undulations Difference ($dN_{Local\ -Model}$) between ($N_{Local\ -Model}$) and (N_{Obs}) for Four Cases in Meters

Case 1 (6) Control Points

Models			Statistical Parameters
Model 1 1^{st} Order	Model 2 2^{nd} Order	Model 3 3^{rd} Order	
0.049	—	—	Max
-0.219	—	—	Min
0.268	—	—	Range
-0.063	—	—	Average
0.097	—	—	STDEV

Case 2 (12) Control Points

Models			Statistical Parameters
Model 1 1^{st} Order	Model 2 2^{nd} Order	Model 3 3^{rd} Order	
0.028	0.120	0.114	Max
-0.167	-0.034	-0.014	Min
0.195	0.154	0.129	Range
-0.045	0.028	0.047	Average
0.072	0.059	0.052	STDEV

Case 3 (18) Control Points

Models			Statistical Parameters
Model 1 1^{st} Order	Model 2 2^{nd} Order	Model 3 3^{rd} Order	
0.045	0.051	0.030	Max
-0.137	-0.101	-0.123	Min
0.182	0.152	0.154	Range
-0.034	-0.020	-0.033	Average
0.062	0.053	0.055	STDEV

Case 4 (24) Control Points

Models			Statistical Parameters
Model 1 1^{st} Order	Model 2 2^{nd} Order	Model 3 3^{rd} Order	
0.054	0.059	0.051	Max
-0.117	-0.083	-0.094	Min
0.170	0.142	0.145	Range
-0.023	-0.010	-0.016	Average
0.057	0.050	0.051	STDEV

When compared against observed undulations, this model produces differences range from 0.059 meter to -0.083 meter, with average of -0.010 meter, and a standard deviation equals ± 0.050 meter. This level of precision exceeds that of all tested GGMs models over the study area.

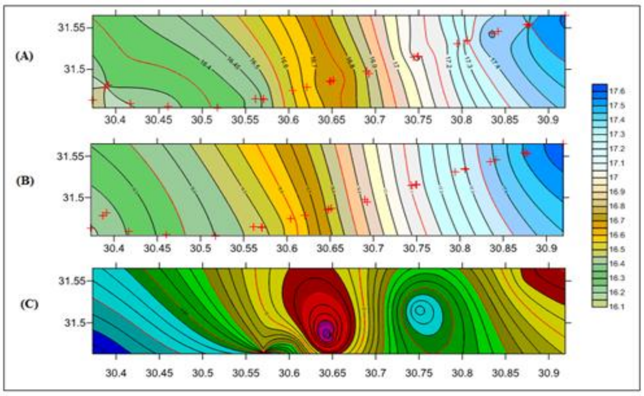

Fig. 9: (A) A Contour Map of Geoid Undulation (NObs), (B) A Contour Map of Geoid Undulation (NLocal_Model), and (C) Contour Map of Difference Geoid Undulations (dNLocal-Model)

VII. CONCLUSION AND RECOMMENDATION

A geoid model is very important to convert the GPS-based ellipsoidal heights into MSL-based orthometric heights usually used in mapping, geomatics, GIS, and surveying applications. So far, there is no national precise geoid model covers the entire Egyptian territories. The current study has investigated GPS/levelling geoid modelling technique and improving its precision through polynomial regression. Additionally, an evaluation of the performance of GGMs in the study area has been carried out. The attained results and concluding remarks can be summarized as following:

- The EGM 2008 represents the most precise global geopotential model to be used for geoid determination in Egypt. it has been found that this model produce geoid undulation differences, in the study area, range from 0.43 meter to 0.691 meter, with an average of 0.57 meter and a standard deviation equals ± 0.06 meter.
- The achievable accuracy of local geoid determination in the study area after using regression method models is ranges between 0.059 meter to -0.083 meter, with an average -0.01 meter and Standard Deviation± 0.05 meter.
- Increasing the number of control points with well spatial distribution, and increasing the polynomial regression order for the same number of control points improves precision of geoid determination model.
- It is highly recommended in future to use GPS/levelling all over Egypt with sufficient and well distributed points to improve the accuracy of geoid determination with cooperation of all surveying authorities in Egypt.
- It is recommended to apply other surface modeling techniques (such as, Kriging, Inverse Distance to a Power, Nearest Neighbor, and Natural Neighbor) in order to check their validity and performance in geoid modeling in Egypt.

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