

On the Metric Dimension of Pendent and Prism Graphs of Dodecahedral Other Embedding

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Abstract:- The findings in the present research paper on the metric dimension of the Dodecahedral Other Embedding (denoted here by G) for pendent and prism graphs are bounded. Further it is concluded that only three vertices chosen appropriately suffice to resolve all the vertices of these graphs for $n \equiv 0 \pmod{4}$, $n \geq 16$, $n \equiv 2 \pmod{4}$, $n \geq 18$ and $n \equiv 3 \pmod{4}$, $n \geq 11$ for pendent and prism graphs respectively and only four vertices chosen appropriately suffice to resolve all the vertices of these graphs for $n \equiv 1 \pmod{4}$, $n \geq 17$.

Keywords:- Metric Dimension, Basis, Resolving Set, Dodecahedral Other Embedding.

I. INTRODUCTION

Open Problem:

Further it can be proved that the metric dimensions of Pendent and Prism graphs may be constant.

A. Notations and preliminary results

Let $G(V, E)$ be a connected graph where V and E represents the vertex and edge Set of G respectively. If $x_1, x_2 \in V(G)$ are the two vertices of connected graph G , if there is an edge between x_1 and x_2 then distance of these two vertices i.e. $d(x_1, x_2) \in V(G)$ described as $d(x_1, x_2)$ and it would be the shortest length or smallest x_1 - x_2 path in the connected graph G . Let $w = \{w_1, w_2, w_3, \dots, w_m\}$ be the set of vertices of G which must be an ordered set i.e. while $x \in V(G)$. Then $r(x/W)$ will be the representation of x with respect to w and it is called m -tuple and is denoted by $(d(x/w_1), d(x/w_2), \dots, d(x/w_m))$. [12- 27] Then " W " is a "Resolving Set" for G , if vertices of G which are distinct have distinct representation with respect to W . [8] A "Basis" for G is actually a set of minimum cardinality and when we take Cardinality of the basis of G then it would be the metric dimension of G written as $\text{Dim}(G)$. For an order set of vertices $w = \{w_1, w_2, w_3, \dots, w_m\}$ of a graph G , The i -th component of $r(x/W)$ is 0 if and only if $x = w_i$. Thus to show that W is resolving set it suffices to verify that $r(x_1/W) \neq r(x_2/W)$ for each pair of distinct points $x_1, x_2 \in V(G)$. A useful property in finding $\text{dim}(G)$ is the following:

B. Lemma : [27]

Let W be the resolving set for a connected graph G and $x_1, x_2 \in V(G)$ if $r(x_1/W) = r(x_2/W)$ for all $w \in V(G) \setminus \{x_1, x_2\}$, Then $\{x_1, x_2\} \cap W \neq \emptyset$.

C. Literature Review:

The first mathematician who introduces the idea or Concept of metric dimension of graphs is Slater (1975) [24 - 25] and after his idea the other researchers in graph theory have attempted the problem of metric dimension of different types of graphs.

[23] Melter and Tomescu (1984) found the metric basis in digital geometry and [25] Yashmanov (1987) found approximations for metric dimension of graphs in terms of diameter and number of vertices. [26] Shanmukhaet al. (2001) offered an analysis on metric dimension of some families of graph and also they determined the actual metric dimension of wheel. [21] Poisson et al. (2002) evaluated the metric dimension of unicyclic graph and they proved that a connected graph is unicyclic if it includes exactly one cycle more over sharp bounds for metric dimension of unicyclic graphs were also established. [9-11] Caceres et al. (2005) established results on minimum resolving sets of certain classes of graphs and examined their behaviour with respect to join and Cartesian product of graphs. [14-15] Imran et al. (2010) studied the metric dimension of some classes of convex polytopes which were obtained by the combinations of two different graphs of convex polytopes. It was shown that these classes of convex polytopes had the constant metric dimension and only three vertices chosen appropriately sufficient to resolve all the vertices of these classes of convex polytopes. [1,2,14-15] Murtaza Ali et al. (2012) the metric dimension of Mobius Ladders and metric dimension of two unique families of Graphs. He also worked on the path related graph having constant metric dimension.

II. THE PENDENT AND PRISM GRAPHS

In this paper we have found and studied the metric dimension of Pendent and Prism graphs. This graph has the following set of vertices and the set of edges denoted by $V(G)$ and $E(G)$ for pendent and prism graphs as under:

$$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$$

And

$$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\}$$

for $1 \leq i \leq n$, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

For our convenience, we represent the cycle induced by $\{u_1, u_2, \dots, u_n\}$ the inner cycle, the cycle induced by $\{v_1, v_2, \dots, v_n\}$ the outer cycle and the set of outer vertices by $\{w_1, w_2, \dots, w_n\}$. Again the vertices choice chosen is crucial for the basis. Note that throughout our discussion remember that $\text{ext}^{(1)}(G)$ and $\text{ext}^{(2)}(G)$ stands for the Pendent and the Prism graphs respectively.

Case 1: $n \equiv 0 \pmod{4}$, $n \geq 16$

In this case it can be written as $n = 4k$, $k \geq 4$, and $k \in \mathbb{Z}^+$. The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\} \subseteq V(G)$, $k \geq 4$.

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Theorem 1:

Prove that the metric dimension $\dim(\text{ext}^{(1)}(G)) \leq 3$ and $\dim(\text{ext}^{(2)}(G)) \leq 3$ for which $n \geq 16$.

Proof:

In this case it can be written as $n = 4k$, $k \geq 4$, and $k \in \mathbb{Z}^+$. The resolving set in general form is $W = \{u_1, u_7, v_{2k+3}\} \subseteq V(G)$, $k \geq 4$.

For the $\text{ext}^{(1)}(G)$ The vertex and the edge sets are as under:

$$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\} \text{ and}$$

$$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\}$$

For $\text{ext}^{(2)}(G)$, Prism graph, The set of vertices and edges are as under:

$$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\} \text{ and}$$

$$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\} \cup \{w_i w_{i+1}\}$$

for $1 \leq i \leq n$ respectively, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

For particular value of $n = 20$ Pendent graph is shown in figure,

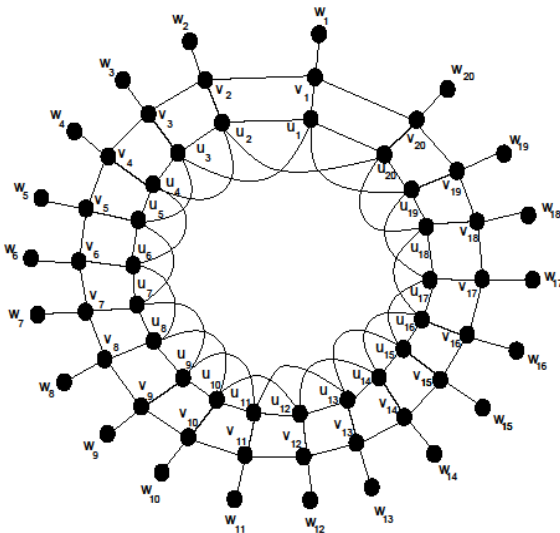


Figure 1: The Pendent graph

A. Representations of vertices w.r.t w in general form are:

Representation of even vertices of Pendent and Prism graph:

$$r(w_{2i}/w) = \begin{cases} (3, 5, k+3) & \text{if } i=1 \\ (4, 4, k+3) & \text{if } i=2 \\ (5, 3, k+2) & \text{if } i=3 \\ (i+2, i-1, k-i+5) & \text{if } 4 \leq i \leq k-1 \\ (k-1, 4) & \text{if } i=k \\ (k+2, k, 2) & \text{if } i=k+1 \\ (k+1, k+1, 2) & \text{if } i=k+2 \\ (k, k+2, 4) & \text{if } i=k+3 \\ (2k-i+3, 2k-i+6, i-k+2) & \text{if } k+4 \leq i \leq 2k \end{cases}$$

For particular value of $n = 20$ Prism graph is shown in figure,

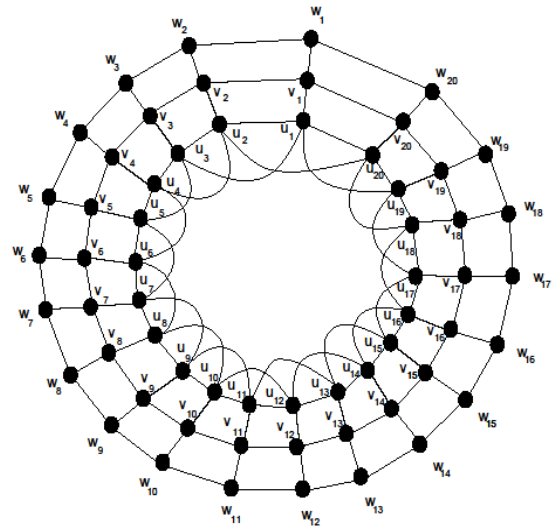


Figure 2: The Prism graph

Representation of odd vertices of Pendent and Prism graph:

$$r(w_{2i-1}/w) = \begin{cases} (2, 5, k+2) & \text{if } i=1 \\ (3, 4, k+3) & \text{if } i=2 \\ (4, 3, k+2) & \text{if } i=3 \\ 2, k-i+5 & \text{if } 4 \leq i \leq k \\ (k+2, k-1, 3) & \text{if } i=k+1 \\ (k+1, k, 1) & \text{if } i=k+2 \\ (k, k+1, 3) & \text{if } i=k+3 \\ (2k-i+3, 2k-i+6, i-k+1) & \text{if } k+4 \leq i \leq 2k \end{cases}$$

Case 2: $n \equiv 1 \pmod{4}$, $n \geq 17$

in general form it can be written as $n = 4k+1$, $k \geq 4$ for $k \in \mathbb{Z}^+$, and the resolving set in general form is $W = \{u_1, u_2, u_7, v_{2k+2}\} \subseteq V(G)$, $k \geq 4$.

Theorem 2:

Prove that the metric dimension $\dim(\text{ext}^{(1)}(G)) \leq 4$ and $\dim(\text{ext}^{(2)}(G)) \leq 4$ for which $n \geq 17$.

Proof:

In general form it can be written as $n = 4k+1$, $k \geq 4$ for $k \in \mathbb{Z}^+$, and the resolving set in general form is $W = \{u_1, u_2, u_7, v_{2k+2}\} \subseteq V(G)$, $k \geq 4$.

For $\text{ext}^{(1)}(G)$ and $\text{ext}^{(2)}(G)$, The representation of vertices w.r.t W for some particular value $n=13$ are:

$$r(w_1/W) = (2, 3, 5, 6), r(w_2/W) = (3, 2, 5, 6), r(w_3/W) = (3, 3, 4, 6), r(w_4/W) = (4, 3, 4, 5), r(w_5/W) = (4, 4, 3, 4), r(w_6/W) = (5, 4, 3, 3), r(w_7/W) = (5, 5, 2, 2), r(w_8/W) = (5, 5, 3, 1), r(w_9/W) = (5, 5, 3, 2), r(w_{10}/W) = (4, 5, 4, 3), r(w_{11}/W) = (4, 4, 4, 4), r(w_{12}/W) = (2, 3, 4, 4), r(w_{13}/W) = (3, 3, 5, 6).$$

For particular value of $n = 21$, Pendent graph is shown in figure,

B. Representations of vertices w.r.t w in general form are:

Representation of even vertices of Pendent and Prism graph:

$$r(w_{2i}/w) = (k \begin{cases} (3, 2, 5, k+3) & \text{if } i=1 \\ (4, 3, 4, k+2) & \text{if } i=2 \\ (5, 4, 3, k+1) & \text{if } i=3 \\ (i+2, i+1, i-1, k-i+4) & \text{if } 4 \leq i \leq k-1 \\ +2, k+1, k-1, 3) & \text{if } i=k \\ (k+2, k+2, k, 1) & \text{if } i=k+1 \\ (k+1, k+2, k+1, 3) & \text{if } i=k+2 \\ (k, k+1, k+2, 5) & \text{if } i=k+3 \\ (2k-i+3, 2k-i+4, 2k-i+6, i-k+2) & \text{if } k+4 \leq i \leq 2k \end{cases}$$

Representation of odd vertices of Pendent and Prism graph:

$$r(w_{2i-1}/w) = (k \begin{cases} (2, 3, 5, k+3) & \text{if } i=1 \\ (3, 3, 4, k+3) & \text{if } i=2 \\ (4, 4, 3, k+2) & \text{if } i=3 \\ (i+1, i+1, i-2, k-i+5) & \text{if } 4 \leq i \leq k-1 \\ (k+1, k+1, k-2, 4) & \text{if } i=k \\ +2, k+2, k-1, 2) & \text{if } i=k+1 \\ (k+2, k+2, k, 2) & \text{if } i=k+2 \\ (k+1, k+1, k+1, 4) & \text{if } i=k+3 \\ +3 \\ (k, k, k+2, 6) & \text{if } i=k+4 \\ (2k-i+4, 2k-i+4, 2k-i+7, i-k+2) & \text{if } k+5 \leq i \leq 2k+1 \end{cases}$$

Case 3: $n = 2 \pmod{4}$, $n \geq 18$

In this case it can be written as $n = 4k + 2$, $k \geq 4$ for $k \in \mathbb{Z}^+$,

The resolving set

in general form is $W = \{u_1, u_4, v_{2k+4}\} \subseteq V(G)$, $k \geq 4$.

For some particular value $n=13$, The representation of vertices w.r.t W of $\text{ext}^{(1)}(G)$ and $\text{ext}^{(2)}(G)$ are:

$$\begin{aligned} r(w_1/W) &= (2, 4, 7), r(w_2/W) = (3, 3, 7), r(w_3/W) = (3, 3, 8), r(w_4/W) = (4, 2, 7), r(w_5/W) = (4, 3, 7), r(w_6/W) = (5, 3, 6), \\ r(w_7/W) &= (5, 4, 6), r(w_8/W) = (6, 4, 5), r(w_9/W) = (6, 5, 4), r(w_{10}/W) = (7, 5, 3), r(w_{11}/W) = (6, 6, 2), \\ r(w_{12}/W) &= (6, 6, 1), r(w_{13}/W) = (5, 7, 2), r(w_{14}/W) = (5, 6, 3), r(w_{15}/W) = (4, 6, 4), r(w_{16}/W) = (4, 5, 5), r(w_{17}/W) = (3, 5, 6), \\ r(w_{18}/W) &= (3, 4, 6). \end{aligned}$$

Theorem 3:

Prove that the metric dimension $\dim(\text{ext}^{(1)}(G)) \leq 3$ and $\dim(\text{ext}^{(2)}(G)) \leq 3$ for which $n \geq 18$.

Proof:

In this case it can be written as $n = 4k + 2$, $k \geq 4$, and $k \in \mathbb{Z}^+$, The resolving set in general form is $W = \{u_1, u_4, v_{2k+4}\} \subseteq V(G)$, $k \geq 4$.

For the $\text{ext}^{(1)}(G)$ The vertex and the edge sets are as under:

$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$ and

$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\}$

For $\text{ext}^{(2)}(G)$, Prism graph, The set of vertices and edges are as under:

$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$

and

$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\} \cup \{w_i w_{i+1}\}$

for $1 \leq i \leq n$ respectively, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively.

For particular value of $n = 22$, Pendent graph is shown in figure,

C. Representations of vertices w.r.t w in general form are:

Representation of even vertices of Pendent and Prism graphs:

$$r(w_{2i}/w) = (k \begin{cases} (3, 3, k+3) & \text{if } i=1 \\ (i+2, i, k-i+5) & \text{if } 2 \leq i \leq k \\ +3, k+1, 3) & \text{if } i=k+1 \\ (k+2, k+2, 1) & \text{if } i=k+2 \\ (k+1, k+1, 3) & \text{if } i=k+3 \\ (2k-i+4, 2k-i+5, i-k+1) & \text{if } k+4 \leq i \leq 2k+1 \end{cases}$$

Representation of odd vertices of Pendent and Prism graphs:

$$r(w_{2i-1}/w) = (k \begin{cases} (2, 4, k+3) & \text{if } i=1 \\ (3, 3, k+4) & \text{if } i=2 \\ (i+1, i, k-i+6) & \text{if } 3 \leq i \leq k \\ +2, k+1, 4) & \text{if } i=k+1 \\ (k+2, k+2, 2) & \text{if } i=k+2 \\ (k+1, k+3, 2) & \text{if } i=k+3 \\ (k, k+2, 4) & \text{if } i=k+4 \\ (2k-i+4, 2k-i+6, i-k+1) & \text{if } k+5 \leq i \leq 2k \end{cases}$$

Case 4: $n = 3 \pmod{4}$, $n \geq 11$

In general form it can be written as $n = 4k+3$, $k \geq 4$ and $k \in \mathbb{Z}^+$, The Resolving set in general form is $W = \{u_1, u_4, v_{2k+3}\} \subseteq V(G)$, $k \geq 2$ and $k \in \mathbb{Z}^+$.

For some particular value $n=13$, The representation of vertices w.r.t W of $\text{ext}^{(1)}(G)$ and $\text{ext}^{(2)}(G)$ are:

$r(w_1/W) = (2, 4, 6)$, $r(w_2/W) = (3, 3, 6)$, $r(w_3/W) = (3, 3, 5)$, $r(w_4/W) = (4, 2, 4)$, $r(w_5/W) = (4, 3, 3)$, $r(w_6/W) = (5, 3, 2)$, $r(w_7/W) = (5, 4, 1)$, $r(w_8/W) = (4, 4, 2)$, $r(w_9/W) = (4, 5, 3)$, $r(w_{10}/W) = (3, 5, 4)$, $r(w_{11}/W) = (3, 4, 5)$.

Theorem 4:

Prove that the metric dimension $\dim(\text{ext}^{(1)}(G)) \leq 3$ and $\dim(\text{ext}^{(2)}(G)) \leq 3$ for which $n \geq 11$.

Proof:

In this case the Resolving set in general form is $W = \{u_1, u_4, v_{2k+3}\} \subseteq V(G)$, $k \geq 2$ and $k \in \mathbb{Z}^+$.

For the $\text{ext}^{(1)}(G)$ The vertex and the edge sets are as under:

$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$ and

$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\}$

For $\text{ext}^{(2)}(G)$, Prism graph, The set of vertices and edges are as under:

$V(G) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n, w_1, w_2, \dots, w_n\}$ and

$E(G) = \{u_i u_{i+2}, u_i v_i\} \cup \{v_i v_{i+1}\} \cup \{v_i w_i\} \cup \{w_i w_{i+1}\}$ for $1 \leq i \leq n$ respectively, where the indices $n+1$ and $n+2$ must be replaced by 1 and 2 respectively. For particular value of $n = 19$, Pendent graph is shown in figure,

For particular value of $n = 19$, Prism graph is shown in figure,

D. Representations of vertices w.r.t w in general form are:

Representation of even vertices of Pendent and Prism graphs:

$$r(w_{2i}/w) = (k) \begin{cases} (3, 3, k+4) & \text{if } i=1 \\ (i+2, i, k-i+5) & \text{if } 2 \leq i \leq k-1 \\ (k+2, k, 4) & \text{if } i=k \\ +3, k+1, 2) & \text{if } i=k+1 \\ (k+2, k+2, 2) & \text{if } i=k+2 \\ (k+1, k+3, 4) & \text{if } i=k+3 \\ (2k-i+4, 2k-i+6, i-k+2) & \text{if } k+4 \leq i \leq k+1 \end{cases}$$

Representation of even vertices of Pendent and Prism graphs:

$$r(w_{2i-1}/w) = (k) \begin{cases} (2, 4, k+4) & \text{if } i=1 \\ (3, 3, k+3) & \text{if } i=2 \\ (i+1, i, k-i+5) & \text{if } 3 \leq i \leq k \\ +2, k+1, 3) & \text{if } i=k+1 \\ (k+3, k+2, 1) & \text{if } i=k+2 \\ (k+2, k+3, 3) & \text{if } i=k+3 \\ (2k-i+5, 2k-i+6, i-k+1) & \text{if } k+4 \leq i \leq 2k+2 \end{cases}$$

III. CONCLUSION

The purpose of this paper was to find the metric dimension of Dodecahedral Other Embedding by using the technique of shortest distance, showing that the distinct vertices has distinct representation with respect to the resolving set W.

The resolving set W is a set of vertices which is subset of the set of vertices of the graph G denoted by V(G). Keeping in view the very fact that no two vertices of G has same representation with respect to the resolving set W. We have found the metric dimension for Pendent and the Prism graphs of Dodecahedral Other Embedding. We have also observed that the metric dimension of all these graphs is Bounded and $n \equiv 1 \pmod{4}$ for $n \geq 17$ for Pendent and the Prism graphs of Dodecahedral Other Embedding which is bounded above by 4. Note that only four vertices are appropriately chosen suffice to resolve all the vertices of these graphs except for $n \equiv 0 \pmod{4}$ for $n \geq 16$ for this graph and for $n \equiv 2 \pmod{4}$ for $n \geq 18$ and $n \equiv 3 \pmod{4}$ for $n \geq 11$ for Pendent and the Prism graphs of Dodecahedral Other Embedding for which only three vertices are appropriately chosen suffices to resolve all the vertices of these graphs for which the metric dimension of these cases is bounded by 3.

We have proved that the metric dimension of G is bounded for Pendent and the Prism graphs of Dodecahedral Other Embedding as given below:

- $\dim(G) \leq 3$ for $n \equiv 0 \pmod{4}$ and $n \geq 16$
- $\dim(G) \leq 4$ for $n \equiv 1 \pmod{4}$ and $n \geq 17$
- $\dim(G) \leq 3$ for $n \equiv 2 \pmod{4}$ and $n \geq 18$
- $\dim(G) \leq 3$ for $n \equiv 3 \pmod{4}$ and $n \geq 11$

According to results of this paper the metric dimension is bounded for all cases of for Pendent and the Prism graphs of Dodecahedral Other Embedding and there is an open problem for constant metric dimension of these cases.

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