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Star-in-Coloring of Arbitrary Super Subdivision of Graphs and the Splitting Graphs

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Abstract*—*In this paper, we have shown that the arbitrary supersubdivision of a path, a cycle, a fan graph, a wheel graph, a helm graph and a gear graph by a complete bi-partite graph $K_{2,m}$ for any m admits star-in-coloring. In addition, we have proved the fan graph and the splitting graph of a path, a cycle and a fan graph also admit star-in-coloring. 2000 Mathematics Subject Classification: 05C15, 05C20.

Keywords: star-in-coloring, splitting graph, fan graph, wheel graph, helm graph, gear graph.

I. INTRODUCTION

Sethuraman et al.[1] have introduced the concept of supersubdivision of edges by the complete bi-partite graph and they discussed the supersubdivision of a path and a cycle. Sethuraman et al.[1] states that for any $n \geq 3$, there exists a supersubdivision of C_n which is graceful. But we found that the arbitrary supersubdivision of a cycle C_n by $K_{2,m}$ fails for some cases. Sudha et al.[2] have found the conditions for the gracefulness of the supersubdivision of a cycle. Sudha et al.[3], [4] have proved the graceful labeling of arbitrary supersubdivision of a helm, centipede, ladder and wheel graphs. The splitting graph of a graph was defined by Sampathkumar et al.[5]. Sudha et al.[6] have proved graceful labeling of the splitting graph of a star graph. In 1973, Grünbaum[7] introduced acyclic coloring and noted the condition that the union of any two color classes induce a forest which can be generalized as bi-partite graphs and calls such type of coloring as star-coloring. Sudha et al.[8], [9], [10] gave a definition for star-in-coloring by combining the conditions of both star-coloring and in-coloring. Sudha et al.[10] have proved the star-in-coloring of splitting graph of a complete bi-partite graph $K_{m,n}$.

In this paper, we have obtained the star-in-coloring chromatic number of the following graphs:
(i) arbitrary supersubdivision of a path,
(ii) arbitrary supersubdivision of a cycle,
(iii) arbitrary supersubdivision of a fan graph,
(iv) arbitrary supersubdivision of a wheel graph,
(v) arbitrary supersubdivision of a helm graph,
(vi) arbitrary supersubdivision of a gear graph,
(vii) fan graph,
(viii) splitting graph of a path,
(ix) splitting graph of a cycle
and (x) splitting graph of a fan graph.

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Definition 1:
Let G be a graph with p vertices and q edges. A graph H is said to be a supersubdivision of G if H is obtained by replacing each and every edge of G by a complete bi-partite graph $K_{2,m}$ for any m.

Definition 2:
For any graph G, the splitting graph is obtained by adding to each vertex v, a new vertex v', so that v' is adjacent to each and every vertex that is adjacent to v in G.

Definition 3:
The join $K_1 V P_n$ of a single vertex K_1 and the path P_n is called a fan graph $(F_{1,n})$. The vertex K_1 is called the core and the edges incident with this core are the spokes.

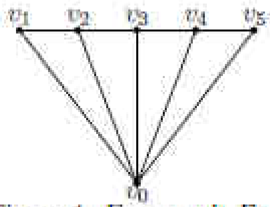

Figure 1. Fan graph $F_{1,5}$.

Definition 4:
A wheel graph (W_n) of order n, sometimes called as n-wheel, is the join of a vertex K_1 with the cycle C_{n-1}. Normally, the vertex K_1 is placed inside the cycle C_{n-1}. It consists of n vertices and $2(n-1)$ edges. The inner edges here are also called spokes.

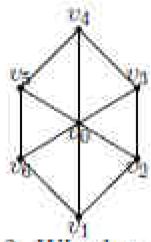

Figure 2. Wheel graph W_7.

Definition 5:
If each and every vertex of the outer cycle of a wheel graph (W_n) has an edge with a new vertex, then it is a helm graph (H_n).

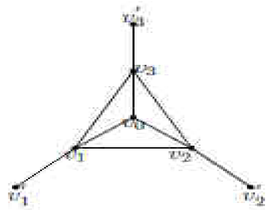

Figure 3. Helm graph H_4.

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Definition 6:
The gear graph, also sometimes known as a bi-partite wheel graph, is a wheel graph by the supersubdivision of each edge of the outer cycle by $K_{2,1}$ and is denoted by $G_{1,n}$. The graph $G_{1,n}$ has $n+1$ vertices and $\left(\frac{3n}{2}\right)$ edges.

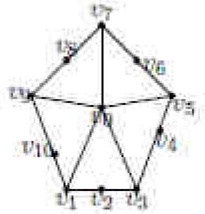
Figure 4. Gear graph $G_{1,10}$.

Many authors like Ma et al.[11] discussed about the gear graph and they gave the representation G_n for gear graph with $2n+1$ vertices. But we have given the definition for gear graph using the concept of supersubdivision.

Definition 7:
A star-coloring of a graph G is a proper coloring of a graph with the condition that no path on four vertices is bi-colored.

Definition 8:
An in-coloring of a digraph G is a proper coloring of the underlying graph G if for any path P_3 of length 2 with the end vertices of the same color are oriented towards the central vertex.

Definition 9:
A graph G is said to be star-in-colored if
1. no path on four vertices is bi-colored
2. any path of length 2 with end vertices of same color are directed towards the middle vertex.

The minimum number of colors required to color a graph G satisfying the above conditions for star-in-coloring is called the star-in-coloring chromatic number of G and is denoted by $\chi_{si}(G)$.

Illustration 1:

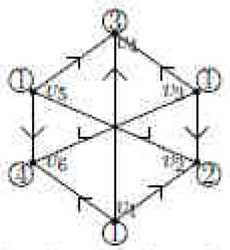
Figure 5. Star-in-coloring of a graph, G.

Consider the graph as shown in fig.5. The vertices v_1, v_3, v_5 are assigned with color 1 and the vertices v_2, v_4, v_6 are assigned with the colors 2, 3 and 4 respectively. This pattern of coloring satisfies the definition of star-in-coloring. It should be noted that in this graph no two adjacent vertices have the same color and no path on four vertices is bi-colored; each and every edge in a path of length two in which the end vertices have the same color are oriented towards the central vertex.

II. STAR-IN-COLORING OF ARBITRARY SUPERSUBDIVISION OF GRAPHS

Theorem 1:
Arbitrary supersubdivision of a path $P_n (n \geq 2)$ by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 3 for all n.

Proof:
Consider a path, P_n with n vertices and $n-1$ edges. The vertices are denoted by v_i, $1 \leq i \leq n$. By the definition-1 each and every edge $v_i v_{i+1}$, $1 \leq i < n$ of a path P_n is replaced by a complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We obtain a new graph with additional vertices u_{ij} in between v_i and v_{i+1} where $1 \leq i \leq n-1$ and $1 \leq j \leq m$.
If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\ldots\}$ such that
$f: V \rightarrow \{1,2,3,\ldots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

$$f(v_i) = \begin{cases} 2, & if\ i \equiv 1 (mod\ 2) \\ 3, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

$$f(u_{ij}) = 1, for\ 1 \leq i \leq n-1; 1 \leq j \leq m$$

We need only three colors for star-in-coloring.
Thus the star-in-coloring chromatic number of the supersubdivision of the path P_n is 3.

Illustration 2:
The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6$ of a path P_6 by the complete bi-partite graphs $K_{2,3}, K_{2,4}, K_{2,2}, K_{2,3}$ and $K_{2,5}$ respectively is shown in figure-6(b). As per theorem-1 the graph is star-in-colored.


Figure 6(a). Path P_6.

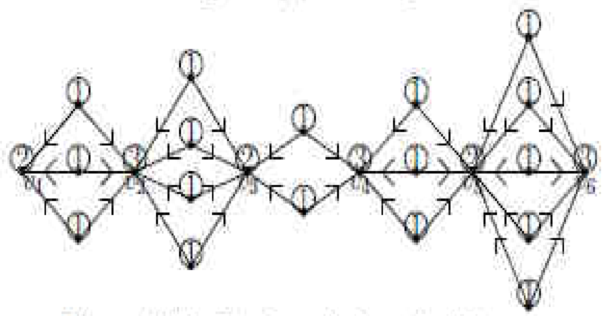
Figure 6(b). Star-in-coloring of arbitrary supersubdivision of a path P_6.

The star-in-coloring chromatic number of the supersubdivision of the path P_6 is 3.

Theorem 2:
Arbitrary supersubdivision of a cycle C_n $(n > 2)$ by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 3 for even n and 4 for odd n.

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Proof:

The vertices of the cycle C_n be denoted by v_i, $1 \leq i \leq n$.The edges of C_n are replaced by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) by definition-1. These newly added vertices between v_i and v_{i+1} be denoted by u_{ij} for $1 \leq i \leq n-1$ and $1 \leq j \leq m$ and the vertices between v_n and v_1 be denoted by u_{nj} for $1 \leq j \leq m$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\ldots\}$ such that

$f: V \to \{1,2,3,\ldots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

There are two cases (i) n odd and (ii) n even.

Case (i): n odd

The vertices v_i's of the cycle C_n are colored as

$$f(v_i) = \begin{cases} 2, & if\ i \equiv 1(mod\ 2) and\ 1 \leq i < n \\ 3, & if\ i \equiv 0(mod\ 2) \\ 4, & if\ i = n \end{cases}$$

The newly added vertices u_{ij} are colored as

$$f(u_{ij}) = 1\ for\ 1 \leq i \leq n; 1 \leq j \leq m.$$

The cycle C_n with this arbitrary supersubdivision of edges by the complete bi-partite graphs $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 4.

Case (ii): n even

The vertices v_i's of the cycle C_n are colored as

$$f(v_i) = \begin{cases} 2, & if\ i \equiv 1(mod\ 2) \\ 3, & if\ i \equiv 0(mod\ 2) \end{cases}$$

The newly added vertices u_{ij} are colored as

$$f(u_{ij}) = 1\ for\ 1 \leq i \leq n; 1 \leq j \leq m.$$

The cycle C_n with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 3.

Illustration 3:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1$ of a cycle C_5 by the complete bi-partite graphs $K_{2,3}, K_{2,7}, K_{2,4}, K_{2,2}$ and $K_{2,4}$ respectively is shown in figure-7(b). It admits star-in-coloring by using case(i) of theorem-2.

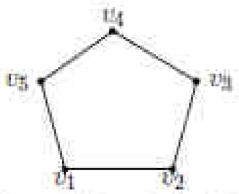

Figure 7(a). Cycle C_5.

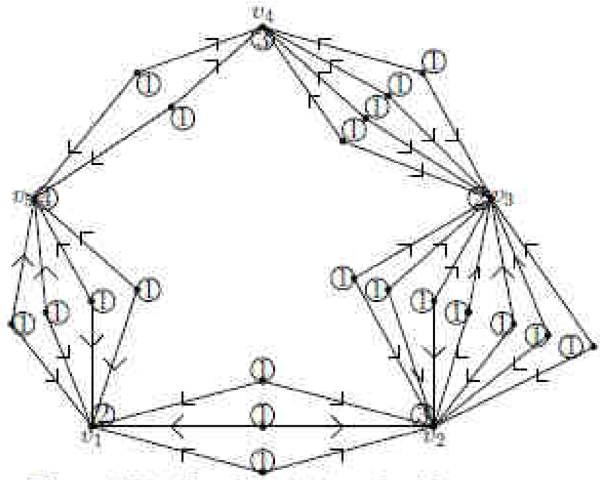
Figure 7(b). Star-in-coloring of arbitrary supersubdivision of a cycle C_5.

The star-in-coloring chromatic number of the supersubdivision of the cycle C_5 is 4.

Illustration 4:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_1$ of a cycle C_6 by the complete bi-partite graphs $K_{2,3}, K_{2,5}, K_{2,4}, K_{2,6}, K_{2,3}$ and $K_{2,2}$ respectively is shown in figure-8(b). It admits star-in-coloring by using case(ii) of theorem-2.

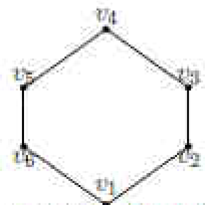

Figure 8(a). Cycle C_6.

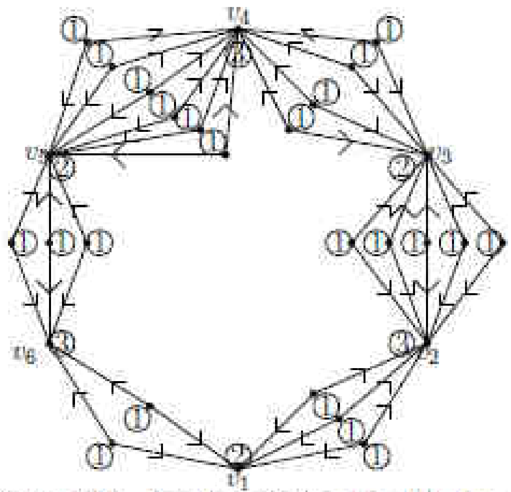
Figure 8(b). Star-in-coloring of arbitrary supersubdivision of a cycle C_6.

The star-in-coloring chromatic number of the supersubdivision of the cycle C_6 is 3.

Theorem 3:

Arbitrary supersubdivision of a fan graph $F_{1,n}(n \geq 2)$ by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 4 for all $n \geq 2$.

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Proof:

A fan graph, $F_{1,n}$ with $n+1$ vertices and $2n-1$ edges has the vertices denoted by $v_i, 1 \le i \le n$ on the path and the core vertex v_0. By the definition-1 each and every edge of the graph is replaced by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We obtain a new graph with additional vertices u_{ij} in between v_i and v_{i+1} where $1 \le i \le n-1$ and $1 \le j \le m$. The newly added vertices between v_0 and $v_i, 1 \le i \le n$ be denoted by $u_{0j}^i, 1 \le j \le m$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\ldots\}$ such that

$f: V \to \{1,2,3,\ldots\}$ such that $f(u) \ne f(v)$ if $uv \in E$

The vertices v_i's of the fan graph $F_{1,n}$ are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

The newly added vertices u_{ij} and u_{0j}^i are colored as

$$f(u_{ij}) = 1, \text{ for } 1 \le i \le n-1 \text{ and for all } j$$

$$f(u_{0j}^i) = 1, \text{ for } 1 \le i \le n \text{ and for all } j$$

The fan graph $F_{1,n}$ with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 4.

Illustration 5:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5$ of a fan graph $F_{1,5}$ by the complete bi-partite graphs $K_{2,2}, K_{2,3}, K_{2,4}, K_{2,3}, K_{2,3}, K_{2,2}, K_{2,3}, K_{2,2}$ and $K_{2,3}$ respectively is shown in figure-9(b). It admits star-in-coloring by using theorem-3.

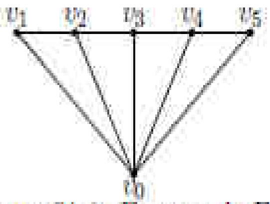

Figure 9(a). Fan graph $F_{1,5}$.

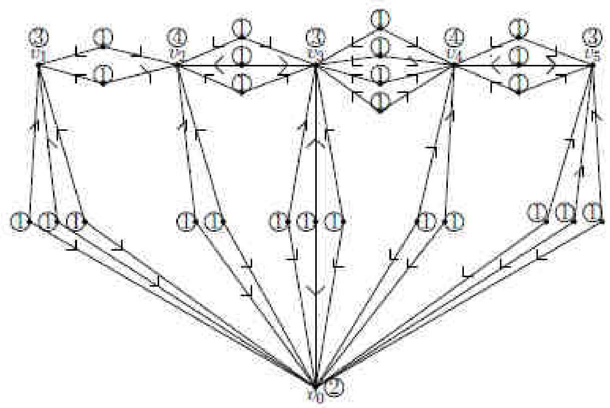
Figure 9(b). Star-in-coloring of arbitrary supersubdivision of fan graph $F_{1,5}$.

The star-in-coloring chromatic number of the supersubdivision of the fan graph $F_{1,5}$ is 4.

Theorem 4:

Arbitrary supersubdivision of a wheel graph W_n $(n > 2)$ by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 5 for even n and 4 for odd n.

Proof:

A wheel graph, W_n with n vertices and $2(n-1)$ edges has the vertices denoted by v_i, $0 \le i \le n-1$. By the definition-1 each and every edge of a wheel graph W_n is replaced by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We obtain a new graph with additional vertices u_{ij} in between v_i and v_{i+1} where $1 \le i \le n-1$ and $1 \le j \le m$. The vertices between v_n and v_1 be denoted by u_{nj} for $1 \le j \le m$ and the vertices between v_0 and v_i, $1 \le i \le n-1$ be denoted by u_{0j}^i, $1 \le j \le m$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\ldots\}$ such that

$f: V \to \{1,2,3,\ldots\}$ such that $f(u) \ne f(v)$ if $uv \in E$

Case (i): For odd n

The vertices v_i's of the wheel graph W_n are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

The newly added vertices u_{ij} and u_{0j}^i are colored as

$$f(u_{ij}) = 1, \text{ for } 1 \le i \le n \text{ and for all } j$$

$$f(u_{0j}^i) = 1, \text{ for } 1 \le i < n \text{ and for all } j$$

The wheel graph W_n with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 4.

Case (ii): For even n

The vertices v_i's of the wheel graph W_n are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) and\ 1 \le i < n-1 \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_{n-1}) = 5$$

The newly added vertices u_{ij} and u_{0j}^i are colored as

$$f(u_{ij}) = 1, \text{ for } 1 \le i \le n \text{ and for all } j$$

$$f(u_{0j}^i) = 1, \text{ for } 1 \le i < n \text{ and for all } j$$

The wheel graph W_n with this arbitrary supersubdivision of edges by the complete bi-partite graphs $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 5.

Illustration 6:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_1, v_0v_1, v_0v_2, v_0v_3, v_0v_4$ of a wheel graph W_5 by the complete bi-partite graphs $K_{2,4}, K_{2,3}, K_{2,3}, K_{2,5}, K_{2,3}, K_{2,4}, K_{2,4}$ and $K_{2,3}$ respectively is shown in figure-10(b). It admits star-in-coloring by using case(i) of theorem-4.

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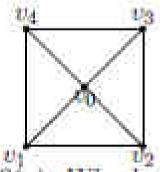

Figure 10(a). Wheel graph W_5.

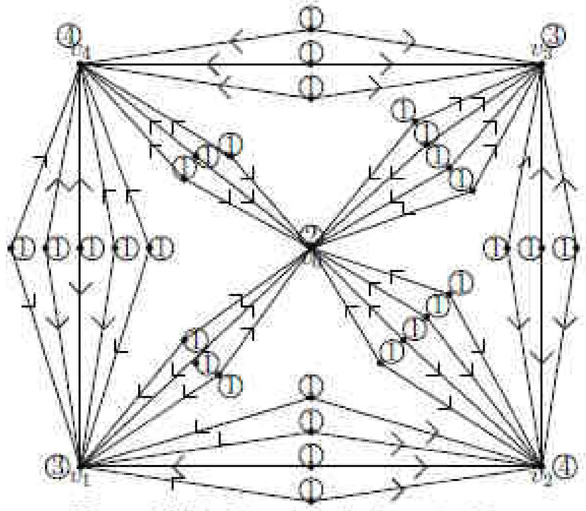

Figure 10(b). Star-in-coloring of arbitrary supersubdivision of a wheel graph W_5.

The star-in-coloring chromatic number of the supersubdivision of the wheel graph W_5 is 4.

Illustration 7:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_1, v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_0v_5$ of a wheel graph W_6 by the complete bi-partite graphs $K_{2,3}, K_{2,3}, K_{2,4}, K_{2,5}, K_{2,3}, K_{2,3}, K_{2,2}, K_{2,4}, K_{2,3}$ and $K_{2,5}$ respectively is shown in figure-11(b). It admits star-in-coloring by using case(ii) of theorem-4.

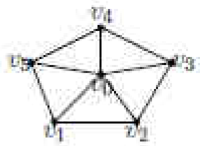

Figure 11(a). Wheel graph W_6.

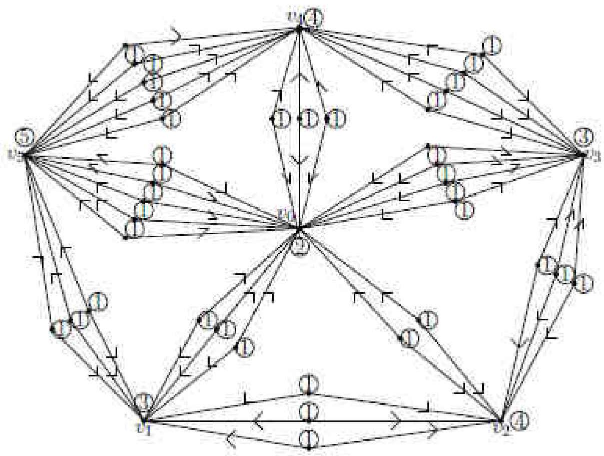

Figure 11(b). Star-in-coloring of arbitrary supersubdivision of a wheel graph W_6.

The star-in-coloring chromatic number of the supersubdivision of the wheel graph W_6 is 5.

Theorem 5:

Arbitrary supersubdivision of a helm graph H_n by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 5 for even n and 4 for odd n.

Proof:

A helm graph, H_n consists of $2n-1$ vertices and $3(n-1)$ edges. Let the central vertex be denoted by v_0 and the vertices on the cycle be denoted by v_i, $1 \le i \le n-1$ and the pendent vertices are denoted by v_i', $1 \le i \le n-1$. By the definition-1 each and every edge of a wheel graph W_n is replaced by a complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We obtain a new graph with additional vertices u_{ij} in between v_i and v_{i+1} for all $1 \le i \le n-1$ and $1 \le j \le m$. The vertices between v_0 and v_i, $1 \le i \le n-1$ be denoted by u_{0j}^i, $1 \le j \le m$ and u_{ij}' be the additional vertices in between v_i and v_i' for all $1 \le i \le n-1$ and $1 \le j \le m$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \to \{1,2,3,\dots\}$ such that $f(u) \ne f(v)$ if $uv \in E$

Case (i): For even n

The vertices v_i's of the helm graph H_n are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) and\ 1 \le i < n-1 \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_{n-1}) = 5$$

$$f(v_i') = 2, for\ 1 \le i \le n-1$$

The newly added vertices u_{ij}, u_{0j}^i and u_{ij}' are colored as

$f(u_{ij}) = 1$, for $1 \le i \le n-1; 1 \le j \le m$

$f(u_{0j}^i) = 1$, for $1 \le i \le n-1; 1 \le j \le m$

$f(u_{ij}') = 1$, for $1 \le i \le n-1; 1 \le j \le m$

The helm graph H_n with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 5.

Case (ii): For odd n

The vertices v_i's of the helm graph H_n are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

$$f(v_i') = 2, for\ 1 \le i \le n-1$$

The newly added vertices u_{ij}, u_{0j}^i and u_{ij}' are colored as

$f(u_{ij}) = 1$, for $1 \le i \le n-1; 1 \le j \le m$

$f(u_{0j}^i) = 1$, for $1 \le i \le n-1; 1 \le j \le m$

$f(u_{ij}') = 1$, for $1 \le i \le n-1; 1 \le j \le m$

The helm graph H_n with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 4.

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Illustration 8:

The supersubdivision of the edges v_1v_2, v_2v_3, v_3v_1, $v_0v_1, v_0v_2, v_0v_3, v_1v_1', v_2v_2', v_3v_3'$ of a helm graph H_4 by the complete bi-partite graphs $K_{2,5}, K_{2,4}, K_{2,3}, K_{2,2}, K_{2,3}$, $K_{2,3}, K_{2,3}, K_{2,4}$ and $K_{2,5}$ respectively is shown in figure-12(b). It admits star-in-coloring by using case(i) of theorem-5.

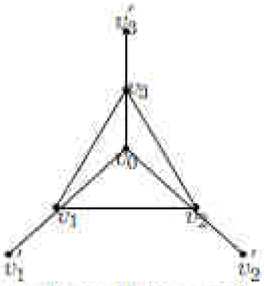

Figure 12(a). Helm graph H_4.

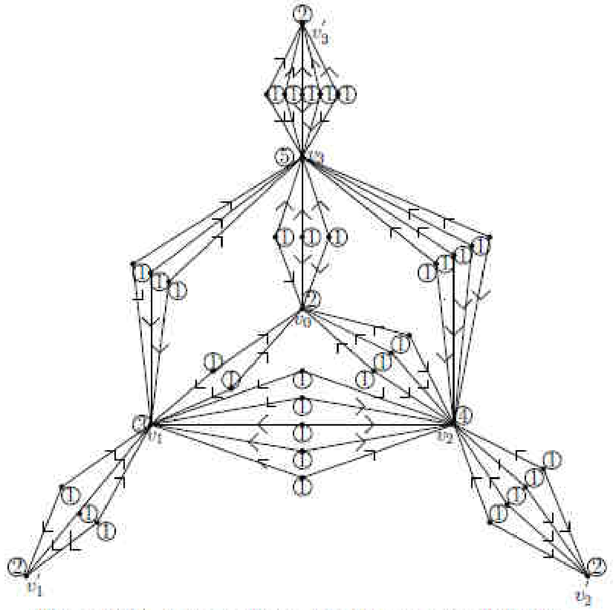

Figure 12(b). Star-in-coloring of arbitrary supersubdivision of a helm graph H_4.

The star-in-coloring chromatic number of the supersubdivision of the helm graph H_4 is 5.

Illustration 9:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_1$, $v_0v_1, v_0v_2, v_0v_3, v_0v_4, v_1v_1', v_2v_2', v_3v_3', v_4v_4'$ of a helm graph H_5 by the complete bi-partite graphs $K_{2,5}, K_{2,2}, K_{2,4}$, $K_{2,3}, K_{2,3}, K_{2,2}, K_{2,4}, K_{2,2}, K_{2,3}, K_{2,4}, K_{2,3}$ and $K_{2,5}$ respect-ively is shown in figure-13(b). It admits star-in-coloring by using case(ii) of theorem-5.

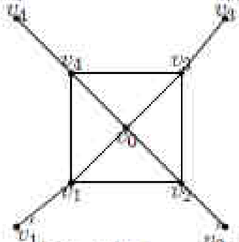

Figure 13(a). Helm graph H_5.

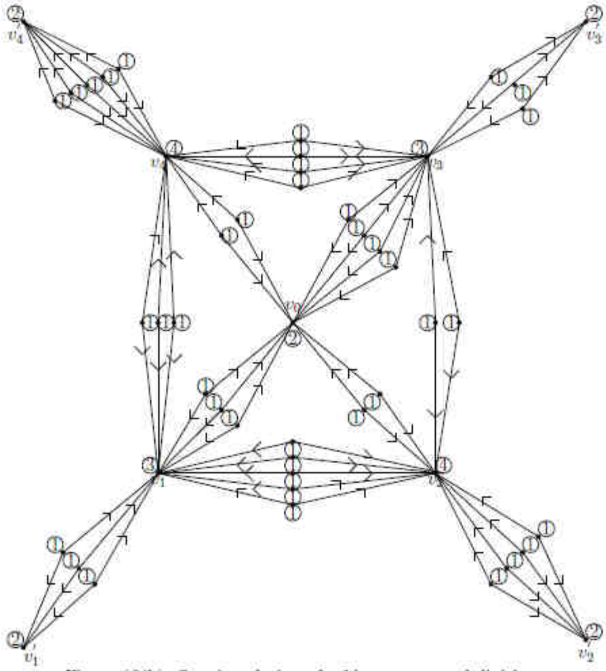

Figure 13(b). Star-in-coloring of arbitrary supersubdivision of helm graph H_5.

The star-in-coloring chromatic number of the supersubdivision of the helm graph H_5 is 4.

Theorem 6:

Arbitrary supersubdivision of a gear graph $G_{1,n}$ by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) admits star-in-coloring with the chromatic number 4 for all n.

Proof:

A gear graph, $G_{1,n}$ with $n+1$ vertices and $\left(\frac{3n}{2}\right)$ edges has the vertices denoted by v_i, $0 \le i \le n$. By the definition-1 each and every edge of a gear graph $G_{1,n}$ is replaced by a complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We obtain a new graph with additional vertices u_{ij} in between v_i and v_{i+1} for odd i, $1 \le i \le n$ and $1 \le j \le m$. The vertices between v_n and v_1 be denoted by u_{nj} for $1 \le j \le m$ and the vertices between v_0 and v_i, odd i, $1 \le i \le n-1$ be denoted by u_{0j}^i, $1 \le j \le m$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \to \{1,2,3,\dots\}$ such that $f(u) \ne f(v)$ if $uv \in E$

The vertices v_i's of the gear graph $G_{1,n}$ are colored as

$$f(v_0) = 2$$

$$f(v_i) = \begin{cases} 3, & if\ i \equiv 1 (mod\ 2) \\ 4, & if\ i \equiv 0 (mod\ 2) \end{cases}$$

The newly added vertices u_{ij} and u_{0j}^i are colored as

$f(u_{ij}) = 1$, for $1 \le i \le n$; $1 \le j \le m$

$f(u_{0j}^i) = 1$, for odd i; $1 \le j \le m$

The gear graph $G_{1,n}$ with this arbitrary supersubdivision of edges by the complete bi-partite graph $K_{2,m}$ (m may vary for each edge) is star-in-colored and its star-in-coloring chromatic number is 4.

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Illustration 10:

The supersubdivision of the edges $v_1v_2, v_2v_3, v_3v_4, v_4v_5, v_5v_6, v_6v_7, v_7v_8, v_8v_1, v_0v_1, v_0v_3, v_0v_5, v_0v_7$ of a gear graph $G_{1,8}$ by the complete bi-partite graphs $K_{2,2}, K_{2,3}, K_{2,3}, K_{2,4}, K_{2,4}, K_{2,3}, K_{2,3}, K_{2,2}, K_{2,4}, K_{2,3}, K_{2,3}$ and $K_{2,5}$ respectively is shown in figure-14(b). It admits star-in-coloring by using case(i) of theorem-6.

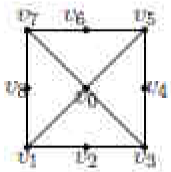

Figure 14(a). Gear graph $G_{1,8}$.

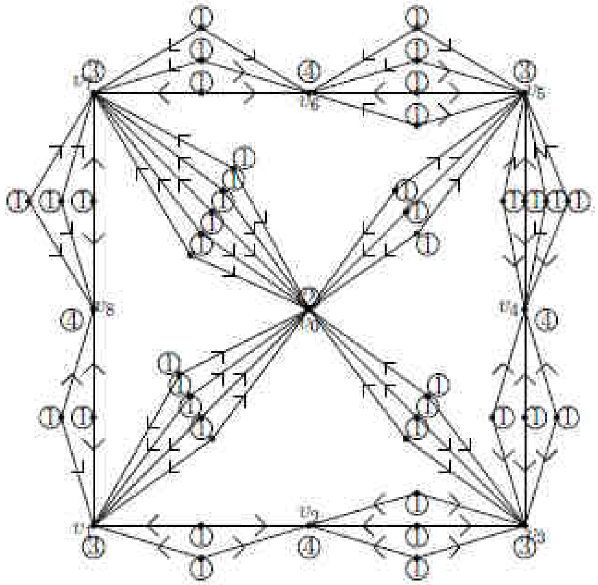

Figure 14(b). Star-in-coloring of arbitrary supersubdivision of a gear graph $G_{1,8}$.

The star-in-coloring chromatic number of the supersubdivision of the gear graph $G_{1,8}$ is 4.

III. STAR-IN-COLORING OF A FAN GRAPH

Theorem 7:

Fan graph $F_{1,n}$ admits star-in-coloring with chromatic number 4 for odd n and $n \geq 9$.

Proof:

Consider a fan graph $F_{1,n}$ which consists of $n+1$ vertices and $2n-1$ edges. The vertices are denoted by v_i, $1 \leq i \leq n$ and v_0 be its central vertex.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \rightarrow \{1,2,3,\dots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

$$f(v_0) = 1$$

$$f(v_i) = \begin{cases} 2, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 3, if\ i \equiv 2(mod\ 4) \\ 4, if\ i \equiv 0(mod\ 4) \end{cases}$$

By using the above definition of f, we can prove that the fan graph admits star-in-coloring.

The star-in-coloring chromatic number of the fan graph is 4.

Illustration 11:

Consider a fan graph $F_{1,9}$. As per the definition-3 it consists of 10 vertices and 17 edges. This graph is star-in-colored by using theorem-7.

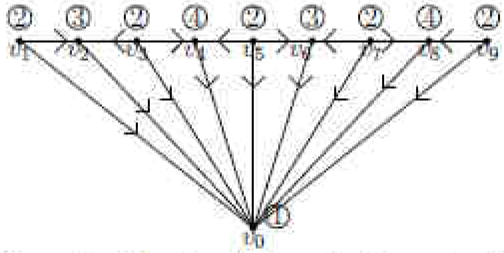

Figure 15. Star-in-coloring of a fan graph $F_{1,9}$.

The star-in-coloring chromatic number of the fan graph $F_{1,9}$ is 4.

IV. STAR-IN-COLORING OF SPLITTING GRAPH OF GRAPHS

Theorem 8:

The splitting graph of a path (P_m) is star-in-colored if its number of edges is even for $m \geq 5$.

Proof:

Consider a path, P_m with m vertices and $m-1$ edges. The vertices are denoted by v_i, $1 \leq i \leq m$. As per the definition of splitting graph we obtain m new vertices v_i', $1 \leq i \leq m$ such that v_i' is adjacent to v_{i+1} and v_{i-1} if there exist edges v_iv_{i+1} and $v_{i-1}v_i$ in the path P_m respectively. The number of vertices present in the newly obtained graph is $2m$ and the number of edges is $3(m-1)$.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \rightarrow \{1,2,3,\dots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

The vertices v_i's of the path P_m are colored as

for $1 \leq i \leq m$,

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 2, if\ i \equiv 2(mod\ 4) \\ 3, if\ i \equiv 0(mod\ 4) \end{cases}$$

The newly added vertices v_i' are colored as

for $1 \leq i \leq m$,

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 4, if\ i \equiv 2(mod\ 4) \\ 5, if\ i \equiv 0(mod\ 4) \end{cases}$$

By using the above definition of f, we can prove that the splitting graph of a path of even length can be star-in-colored.

The star-in-coloring chromatic number of the splitting graph of the path is 5.

Illustration 12:

Consider a path P_9. As per the definition we obtain the splitting graph of a path P_9 which consists of 14 vertices and 18 edges. This graph is star-in-colored by using theorem-8.

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Figure 16(a). Path P_7.

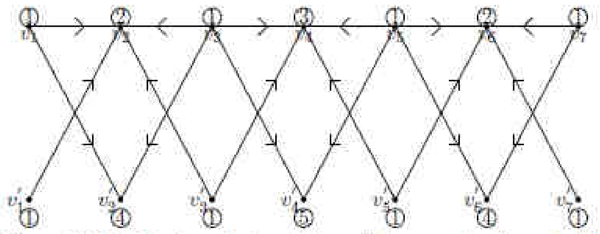

Figure 16(b). Star-in-coloring of the splitting graph of a path P_7.

The star-in-coloring chromatic number of the splitting graph of a path P_9 is 5.

Remark:

A path of length two can be star-in-colored and its star-in-coloring chromatic number is 3.

Theorem 9:

The splitting graph of a cycle (C_n) is star-in-colored if its number of edges is even for $n \geq 4$.

Proof:

Consider a cycle, C_n with n vertices and n edges. The vertices are denoted by v_i, $1 \leq i \leq n$. As per the definition of splitting graph we obtain additional vertices say v_i', $1 \leq i \leq n$ which is adjacent to v_i's according to the definition of splitting graph. The newly obtained graph consists of $2n$ vertices and $3n$ edges.

If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \rightarrow \{1,2,3,\dots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

The vertices v_i's of the cycle C_n are colored in two cases:

Case (i): For $n \equiv 0(mod\ 4), 1 \leq i \leq n$

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 2, if\ i \equiv 2(mod\ 4) \\ 3, if\ i \equiv 0(mod\ 4) \end{cases}$$

The newly added vertices v_i' are colored as

$$f(v_i') = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 4, if\ i \equiv 2(mod\ 4) \\ 5, if\ i \equiv 0(mod\ 4) \end{cases}$$

Case (ii): For $n \equiv 2(mod\ 4), 1 \leq i < n$

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 2, if\ i \equiv 2(mod\ 4)\ and\ 1 \leq i < n \\ 3, if\ i \equiv 0(mod\ 4) \end{cases}$$

$$f(v_n) = 6$$

$$f(v_i') = \begin{cases} 1, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 4, if\ i \equiv 2(mod\ 4)\ and\ 1 \leq i < n \\ 5, if\ i \equiv 0(mod\ 4) \end{cases}$$

$$f(v_n') = 7$$

By using the above definition of f, we can prove that the splitting graph of a cycle C_n can be star-in-colored if cycle is of even length.

The star-in-coloring chromatic number of the splitting graph of a cycle C_n is 5 for $n \equiv 0(mod\ 4)$ and 7 for $n \equiv 2(mod\ 4)$.

Illustration 13:

Consider a cycle C_8. As per the definition-2 it consists of 16 vertices and 24 edges. This graph can be star-in-colored by using case(i) of theorem-9.

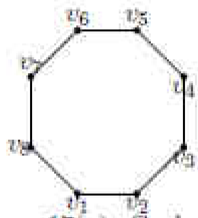

Figure 17(a). Cycle C_8.

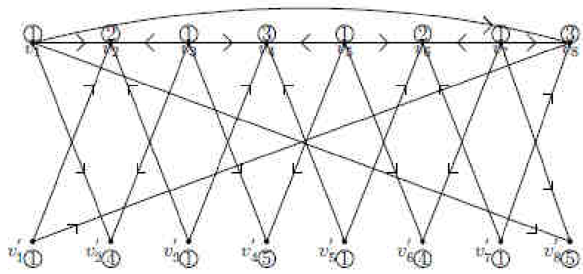

Figure 17(b). Star-in-coloring of the splitting graph of a cycle C_8.

The star-in-coloring chromatic number of the splitting graph of the cycle C_8 is 5.

Illustration 14:

Consider a cycle C_6. As per the definition-2 it consists of 12 vertices and 18 edges. This graph can be star-in-colored by using case(ii) of theorem-9.

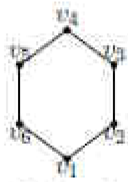

Figure 18(a). Cycle C_6.

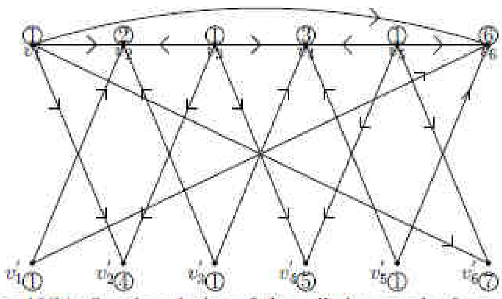

Figure 18(b). Star-in-coloring of the splitting graph of a cycle C_6.

The star-in-coloring chromatic number of the splitting graph of the cycle C_6 is 7.

Theorem 10:

The splitting graph of a fan graph $F_{1,n}$ admits star-in-coloring with the chromatic number 7 for odd n and $n \geq 9$.

Proof:

Consider a fan graph, $F_{1,n}$ with $n+1$ vertices and $2n-1$ edges. The vertices are denoted by v_i, $0 \leq i \leq n$. We obtain additional vertices say v_i', $0 \leq i \leq n$ which is adjacent to v_i's according to the definition-2.

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If V is the vertex set and E is the edge set of the newly obtained graph, define a function f from V to the color set $\{1,2,3,\dots\}$ such that

$f: V \rightarrow \{1,2,3,\dots\}$ such that $f(u) \neq f(v)$ if $uv \in E$

The vertices v_i's of the fan graph $F_{1,n}$ are colored as

$$f(v_o) = 1$$

$$f(v_i) = \begin{cases} 2, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 3, if\ i \equiv 2(mod\ 4) \\ 4, if\ i \equiv 0(mod\ 4) \end{cases}$$

The newly added vertices v_i' are colored as

$$f(v_0') = 5$$

$$f(v_i') = \begin{cases} 2, if\ i \equiv 1(mod\ 4) and\ i \equiv 3(mod\ 4) \\ 6, if\ i \equiv 2(mod\ 4) \\ 7, if\ i \equiv 0(mod\ 4) \end{cases}$$

By using the above definition of f, we can prove that the splitting graph of a fan graph $F_{1,n}$ can be star-in-colored for odd n.

The star-in-coloring chromatic number of the splitting graph of a fan graph is 7.

Illustration 15:

Consider a fan graph $F_{1,9}$. As per the definition-3 it consists of 10 vertices and 17 edges. This graph is star-in-colored by using theorem-10.

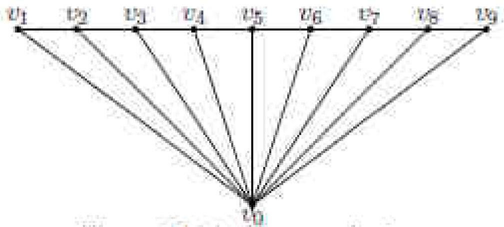

Figure 19(a). Fan graph $F_{1,9}$.

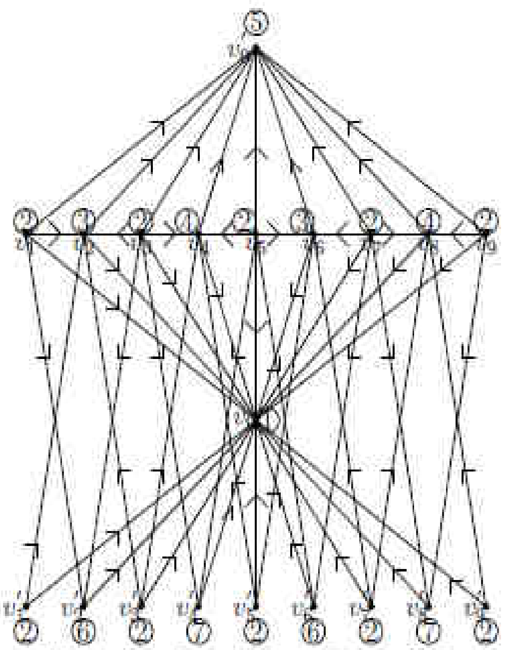

Figure 19(b). Star-in-coloring of the splitting graph of a fan graph $F_{1,9}$.

The star-in-coloring chromatic number of the splitting graph of fan graph $F_{1,9}$ is 7.

V. CONCLUSION

We have discussed and found the star-in-coloring chromatic number of the arbitrary supersubdivision of a path, a cycle, a fan graph, a wheel graph, a helm graph and a gear graph by a complete bi-partite graph $K_{2,m}$ (m may vary for each edge). We have also obtained the star-in-coloring chromatic number of the fan graph and the splitting graph of a path, cycle and fan graph.

Question 1: Is star-in-coloring of the splitting graph of P_m of odd length possible?

Question 2: Is star-in-coloring of the splitting graph of C_n of odd length possible?

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