

DOA Estimation Algorithm for Smart Antennas-An Investigation

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Abstract— High resolution direction-of-arrival (DOA) estimation is important in many applications and over the years many techniques have been proposed. The signal subspace method (MUSIC) [1] has been the most popular and is known to yield asymptotically unbiased and efficient estimates. The MUSIC algorithm estimates the signal subspace from the array measurements and then estimates the parameters of interest from the intersections between the array manifold and the estimated signal. In this work DOA estimation based on MUSIC algorithm and improved MUSIC algorithm is investigated. The classical MUSIC algorithm is analyzed and results of simulations using Matlab are presented. Results for the DOA estimation of the noncoherent signals and coherent signals are found.

Index Terms— MUSIC, Beamforming, direction-of-arrival (DOA), eigen-decomposition.

I. INTRODUCTION

Smart sensor systems are receiving considerable interest recently due to their potential for reducing the interference in communication systems [2]. In smart sensors, Adaptive beamforming (AB) algorithms are used to minimize the signal power of the interference while maximizing the power of the desired signal. If the direction of arrival (DOA) of the desired signal is known, the above problem can be solved using the DOA-based algorithms. Due to steering capability of the beamformer the desired or target signal protected and the effects of the sources of interference are minimized.

In general DOA estimation algorithms are categorized into two groups; the conventional algorithms and the subspace algorithms. Conventional methods for DOA estimation are based on the concepts of beamforming and null steering and do not exploit the statistics of the received signal. In this technique, the DOA of all the signals is determined from the peaks of the output power spectrum obtained from steering the beam in all possible directions. The delay-and-sum method and Capon's minimum variance method are the examples of conventional methods. First one has poor resolution whereas the second one fails when the signals not of interests (SNOIs) are correlated with the signal of interest (SOI). Subspace based methods exploit the orthogonal relationship between the signal subspace and noise subspace to estimate the signal's DOA. Multiple Signal Classification (MUSIC) algorithm and the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) [3] are the two main algorithms fall into this category In MUSIC algorithm Schmidt exploited the property that the desired signal array response is orthogonal to the noise subspace. The signal and noise subspaces are identified using eigen decomposition of the received signal covariance matrix.

Following, the MUSIC spatial spectrum is computed, from which the DOAs are estimated. It is used for the estimation of multiple parameters per source (e.g., azimuth, elevation, range, polarization, etc.) when applied to the array of arbitrary but known configurations and response. This is possible when array response is either measured (calibrated) and stored, or characterized analytically. In addition MUSIC requires a prior information of the second- order spatial statistics of the background noise and interference field.

II. THE SIGNAL MODEL

There are many practical signal processing applications, where data from an array of sensors are collected, and the objective is to locate point sources radiating energy which is detectable by the sensors of the array. These types of problems are mathematically modeled using Green's functions for the particular differential operator that describes the physics of radiation propagation from the sources to the sensors. For intended applications, however, a few reasonable assumptions can be evoked to make the problem analytically tractable.

Schmidt considered the geometry of the signal parameter estimation problem and developed MUSIC doa estimation algorithm in 1977. Until the mid-1970's, direction finding techniques required knowledge of the array directional sensitivity pattern in analytical form, and the task of the antenna designer was to build an array of antennas with the prespecified sensitivity pattern. The designers are relieved from such constraints due to the work done by Schmidt. The reduction in analytical complexity achieved by calibrating the array. Thus, the highly nonlinear problem of calculating the array response to a signal from a given direction was reduced to that of measuring and storing the response. MUSIC extended high resolution DOA estimation to arbitrary arrays of sensors but it could not solve the problem of computational complexity of solutions to the DOA estimation problem.

A) ARRAY RESPONSE VECTOR

Assumed that an antenna array is composed of identical isotropic elements, each element received a time delayed version of the same plane wave with wavelength λ , i. e. each element receives a phase-shifted version of the signal. For example, with a uniform linear array (ULA), the relative phase are also uniformly spaced, with $\psi = (2\pi/\lambda) d \sin \theta$ being the relative phase difference between adjacent elements.

The vector of relative phases is referred to as the steering vector (SV). Array response vector (ARV) is the response of an array to an incident plane wave. It is a combination of the steering vector and the response of each individual element to the incident wave. The general normalized ARV expression for a three-dimensional array of N elements is [4].

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$$\mathbf{a}(\theta, \phi) = \begin{bmatrix} G_1(\theta, \phi)e^{-j\beta r_1} \\ G_2(\theta, \phi)e^{-j\beta r_2} \\ \vdots \\ G_N(\theta, \phi)e^{-j\beta r_N} \end{bmatrix} \dots\dots\dots (1)$$

Where β is the vector wave number of the incident plane wave ($\beta = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta]$ in Cartesian coordinates), $\mathbf{r}_i = [x_i, y_i, z_i]$ is the three dimensional position vector of the i th element in the array and $G_i(\theta, \phi)$ is the gain of the i th element toward the direction (θ, ϕ) , where θ and ϕ are the elevation and azimuth angles, respectively. For an array of isotropic radiators, the ARV simplifies to the SV;

$$\mathbf{a}(\theta, \phi) = [e^{-j\beta r_1}, e^{-j\beta r_2}, \dots e^{-j\beta r_N}]^T \dots\dots\dots (2)$$

B) RECEIVED SIGNAL MODEL

Assuming that K uncorrelated sources transmit signals to an N –element antenna array and the array response for each signal is a function of only one angle parameter (θ) . Well-established narrowband data model is chosen which assumes that as the signal wave fronts propagate across the array, the envelope of the signal is unchanged. Under this model, the received signals can be expressed as a superposition of signals from all the sources and linearly added noise represented by

$$\mathbf{x}(t) = \sum_{k=1}^K \mathbf{a}(\theta_k) s_k(t) + \mathbf{n}(t) \dots\dots\dots (3)$$

Where $\mathbf{x}(t) \in \mathbb{C}^N$ is the complex baseband equivalent received signal vector at the antenna array at time t , or

$$\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_N(t)]^T \dots (4)$$

$s_k(t)$ is the incoming plane wave from the k th source at time t and arriving from the direction θ_k , $\mathbf{a}(\theta_k) \in \mathbb{C}^N$ is the array response vector to this direction, and $\mathbf{n}(t) \in \mathbb{C}^N$ represents additive noise. This complex vector $\mathbf{n}(t)$ is the noise either sensed along with the signals or generated internal to the instrumentation. A single observation $\mathbf{x}(t)$ from the array is termed as a snapshot. In matrix notation, (3) can be written as

$$\mathbf{x}(t) = \mathbf{A}(\Theta)\mathbf{s}(t) + \mathbf{n}(t) \dots\dots\dots (5)$$

Where $\mathbf{A}(\Theta) \in \mathbb{C}^{N \times K}$ is the array response matrix parameterized by the direction of arrival (DOA) (i.e. each column of which represents the array response vector for each signal source), or

$$\mathbf{A}(\Theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_K)] \dots\dots\dots (6)$$

Θ is the vector of all the DOAs, or

$$\Theta = [\theta_1, \theta_2, \dots, \theta_K]^T \dots\dots\dots (7)$$

And $\mathbf{s}(t) \in \mathbb{C}^K$ represents the vector of the incoming signal in amplitude and phase from each signal source at time t , referred as desired signal portion of $\mathbf{x}(t)$, or

$$\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_K(t)]^T \dots\dots\dots (8)$$

The three important features of (5) are that the matrix $\mathbf{A}(\Theta)$ must be time invariant over the observation interval, the model is bilinear in $\mathbf{A}(\Theta)$ and $\mathbf{s}(t)$, and the noise is additive. The majority of algorithms developed for the estimate of the DOAs require that the array response matrix $\mathbf{A}(\Theta)$ be completely known for a given parameter vector Θ . This is usually accomplished by direct calibration in the field, or by analytical means using information about the position and response of each individual sensor.

An element from an unambiguous array manifold of a uniform linear array of identical sensors, is proportional to

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1 \\ e^{j\frac{2\pi}{\lambda}d\sin\theta_1} \\ e^{j\frac{2\pi}{\lambda}2d\sin\theta_2} \\ \vdots \\ e^{j\frac{2\pi}{\lambda}(N-1)d\sin\theta_k} \end{bmatrix} \dots\dots\dots (9)$$

Where λ the wavelength of the impinging wavefront and d is the distance between adjacent elements. For $N \geq K$ an unambiguous $\mathbf{A}(\Theta)$ will be of full-rank K .

For Number of snapshots L where $L > K$, the matrices can be formed as

$$\mathbf{X} = [\mathbf{x}(1), \mathbf{x}(2), \dots, \mathbf{x}(L)] \dots\dots\dots (10)$$

$$\mathbf{S} = [\mathbf{s}(1), \mathbf{s}(2), \dots, \mathbf{s}(L)] \text{ and } \dots (11)$$

$$\mathbf{N} = [\mathbf{n}(1), \mathbf{n}(2), \dots, \mathbf{n}(L)] \dots\dots\dots (12)$$

Where \mathbf{X} and $\mathbf{N} \in \mathbb{C}^{K \times L}$ and $\mathbf{S} \in \mathbb{C}^{K \times L}$, and further we can write

$$\mathbf{X} = \mathbf{A}(\Theta)\mathbf{S} + \mathbf{N} \dots\dots\dots (13)$$

The received signal autocovariance matrix \mathbf{R}_{xx} and the desired signal autocovariance matrix \mathbf{R}_{ss} given by

$$\mathbf{R}_{xx} = \mathcal{E}\{\mathbf{x}(t)\mathbf{x}^H(t)\} \dots\dots\dots (14)$$

$$\mathbf{R}_{ss} = \mathcal{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\} \dots\dots\dots (15)$$

Where H denotes Hermitian matrix operation and $\mathcal{E}\{\cdot\}$ is the expectation operation on the argument. In practical implementation having number of snapshots the expected value would be,

$$\hat{\mathbf{R}}_{xx} \triangleq \lim_{M \rightarrow \infty} \frac{1}{M} \sum_{m=1}^M \mathbf{x}(t_m)\mathbf{x}^H(t_m) \dots\dots\dots (16)$$

The same approximation holds for $\hat{\mathbf{R}}_{ss}$.

The signal and noise vectors are zero mean, and noise is spatially white and statistically independent of the source signals, the different signals are uncorrelated, so, the received signal autocovariance matrix is,

$$\begin{aligned} \mathbf{R}_{xx} &= \mathbf{A}(\Theta)\mathbf{R}_{ss}\mathbf{A}^H(\Theta) + \mathcal{E}\{\mathbf{n}(t)\mathbf{n}^H(t)\} \\ &= \mathbf{A}(\Theta)\mathbf{R}_{ss}\mathbf{A}^H(\Theta) + \sigma_n^2\mathbf{I} \dots\dots\dots (17) \end{aligned}$$

Where σ_n^2 is the noise variance and \mathbf{I} identity matrix.

III. MUSIC ALGORITHM

Inside the algorithm, first general array manifold is defined as the set

$$\mathcal{A} = \{\mathbf{a}(\theta_i : \theta_i \in \Theta)\} \dots\dots\dots (18)$$

For some region Θ of interest. The array manifold is assumed unambiguous and known for all the values of angle θ , either analytically or through some calibration procedure.



The objective is to apply appropriate methods to the received signals so as to extract the region θ out of the range of Θ .

In the absence of noise the observations $\mathbf{x}(t)$ confined entirely to the K - dimensional subspace of \mathbb{C}^K defined by the span of $\mathbf{A}(\Theta)$. In this case determining the DOAs is finding the K unique elements of \mathcal{A} that intersects this subspace. In the presence of noise the observations become full rank.

The approach of MUSIC, and other subspace based methods, is to first estimate the dominant subspace of the observations, and then find the elements of \mathcal{A} that are in some sense close to this subspace.

The subspace estimation is achieved by eigen decomposition of the autocovariance matrix of the received data \mathbf{R}_{xx} . For MUSIC algorithm \mathbf{R}_{ss} is full rank. Using the model in (17) and assuming spatial whiteness, i.e., $\mathcal{E}\{\mathbf{n}(t)\mathbf{n}^H(t)\} = \sigma_n^2 \mathbf{I}$, the eigendecomposition of \mathbf{R}_{xx} gives the eigenvalues λ_n such that $\lambda_1 > \lambda_2 > \lambda_3 > \dots > \lambda_K > \lambda_{K+1} = \lambda_{K+2} = \dots = \lambda_N = \sigma_n^2$ and the corresponding eigenvectors $\mathbf{e}_n \in \mathbb{C}^N$, $n = 1, 2, \dots, N$, of \mathbf{R}_{xx} . \mathbf{R}_{xx} takes the form,

$$\begin{aligned} \mathbf{R}_{xx} &= \sum_{n=1}^N \lambda_n \mathbf{e}_n \mathbf{e}_n^H = \mathbf{E} \mathbf{\Lambda} \mathbf{E}^H \\ &= \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \mathbf{E}_n \mathbf{\Lambda}_n \mathbf{E}_n^H \\ &= \mathbf{E}_s \mathbf{\Lambda}_s \mathbf{E}_s^H + \sigma_n^2 \mathbf{E}_n \mathbf{E}_n^H = \mathbf{E}_s \tilde{\mathbf{\Lambda}}_s \mathbf{E}_s^H + \sigma_n^2 \mathbf{I} \dots (19) \end{aligned}$$

Where $\mathbf{E}_n = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N]$, $\mathbf{E}_s = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_K]$, $\mathbf{E}_n = [\mathbf{e}_{K+1}, \mathbf{e}_{K+2}, \dots, \mathbf{e}_N]$, $\mathbf{\Lambda} = \text{diag}\{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_N\}$, and $\tilde{\mathbf{\Lambda}}_s = \mathbf{\Lambda}_s - \sigma_n^2 \mathbf{I}$. The eigenvectors $\mathbf{E} = [\mathbf{E}_s, \mathbf{E}_n]$ form an orthonormal basis. The span of the K vectors \mathbf{E}_s is signal subspace, and the orthogonal complement spanned by \mathbf{E}_n is noise subspace. DOAs of the desired signals estimated by calculating the MUSIC spatial spectrum over the region of interest:

$$P_{MUSIC}(\theta) = \frac{\mathbf{a}^H(\theta) \mathbf{a}(\theta)}{\mathbf{a}^H(\theta) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta)} \dots \dots \dots (20)$$

Here $\mathbf{a}(\theta)$ s are the array response vectors calculated for all angles θ within the range of interest. As the desired array response vectors $\mathbf{A}(\Theta)$ are orthogonal to the noise subspace, the peaks in the MUSIC spatial spectrum represent the DOA estimates for the desired signals.

Music algorithm is an effective method for direction of arrival, but it can only do with uncorrelated signals. If the condition does not meet bias occurred. In order to solve the problem of the DOA estimation of coherent(correlated) signals spatial smoothing technique is used for enhanced MUSIC algorithm is used [5].in this work improved MUSIC proposed by conjugate reconstruction of the data matrix is investigated [6]

IV. SIMULATION RESULTS

The MUSIC technique for DOA estimation is simulated using MATLAB. Performance of the algorithm has been analyzed as a function of array elements, SNR, no. of snapshots. The simulation is run for two signals coming from different angles 30° and 70° .

1. The MUSIC algorithm for DOA estimation

The first simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, the incident angle is 30° and 70° respectively, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is 30dB, the element spacing is half of the input signal wavelength, array element number is 10 and the number of snapshots is 500. The simulation results are shown in Figure1:

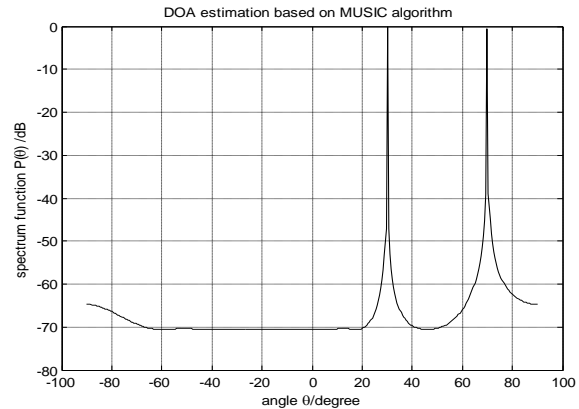


Fig.1 Basic simulation for MUSIC algorithm

As can be seen from Figure1, It may well estimate the number and direction of the incidence signal, which can be used to estimate the independent signal source DOA effectively.

2. MUSIC spectrum for varying number of array elements:

The simulation results are shown in Figure2:

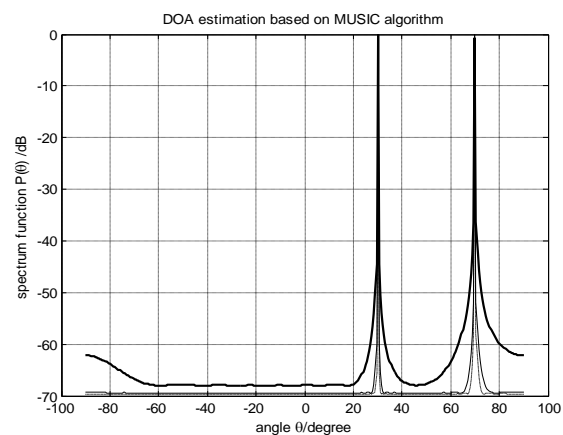


Fig.2. The relationship between MUSIC algorithm and the number of array elements

In figure.2, the dashed line shows the number of array elements is 10, the solid line shows the number of array elements are 50, and the dash-dotted line shows the number of array elements are 100. With other conditions remaining unchanged and with the increase in the number of array elements, DOA estimation spectral beam width becomes narrow and the directivity of the array becomes good, i.e. the ability to distinguish spatial signals is enhanced. Hence, to get more accurate estimations of DOA one can increase the number of array elements, but the more the number of array elements the more the data needs processing; and the more amount of computation, the lower the speed.

3. MUSIC spectrum for varying array element spacing:

The simulation results are shown in fig3 for array spacing $\lambda/6$, $\lambda/2$, and λ . The dashed line shows the estimation when array elements spacing is $\lambda/6$, solid line shows for the spacing $\lambda/2$, and the dash-dotted line shows for the spacing λ . With the other conditions remaining the same, the resolution of MUSIC algorithm improves with the increase in the spacing of array element, but, when the spacing of the array elements is larger than half the wavelength, the estimated spectrum shows false peaks i.e. it lost the estimation accuracy.

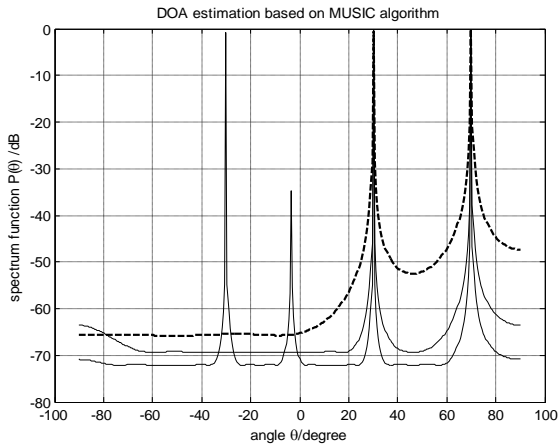


Fig. 3 the relationship between MUSIC algorithm and array element spacing

Hence, in practical applications, more attention should be paid to the spacing of the array elements; element spacing can be increased but must not exceed half the wavelength.

4. MUSIC spectrums for the varying number of snapshots:

With other conditions unchanged and with the increase in the number of snapshots i.e. 5, 50 and 500. DOA estimation simulation results are shown in Figure 4:

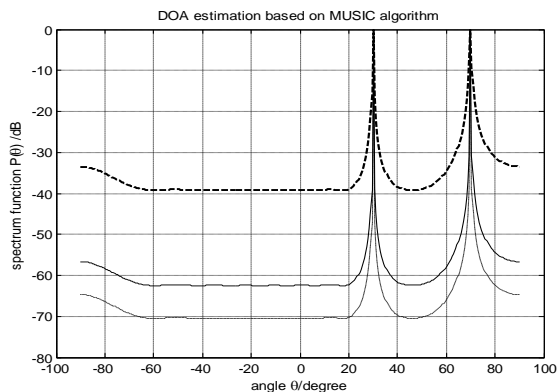


Fig. 4 Relationship between MUSIC algorithm and the number of snapshots

With the increase in the number of snapshots, the beam width of DOA estimation spectrum becomes narrow and the accuracy of MUSIC algorithm increases. Hence, the number of sample snapshots can be expanded to multiply the accuracy of DOA estimation, but the more sample snapshots, the more the data needs to be processed; the more amount of calculation of MUSIC algorithm, the lower the speed. So in practical application, we select reasonable sampling snapshots which ensure the accuracy of DOA

estimation, minimize the amount of computation and accelerating the speed of work and saving resources.

5. MUSIC spectrum and varying SNR:

For $SNR = 30dB, 0dB$ and $30dB$ simulation results are shown in Figure 5:

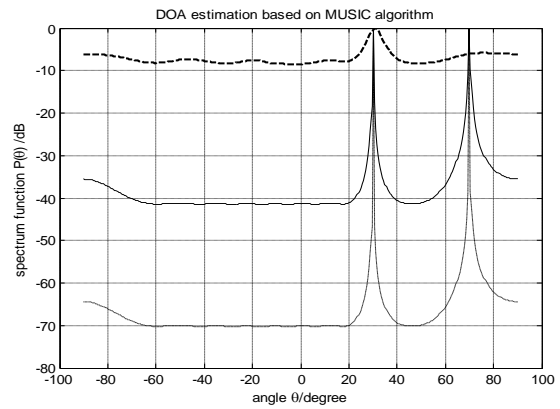


Fig. 5 simulations results for the relationship between MUSIC algorithm and SNR

As can be seen from Figure 5, the dashed line shows the SNR is $-30dB$, the solid line shows the SNR is $0dB$ and the dash-dotted line shows the SNR is $30dB$. With the other conditions remaining unchanged, with the increase in the number of SNR, the beam width of DOA estimation spectrum becomes narrow, the direction of the signal becomes clearer, and the accuracy of MUSIC algorithm is also increased.

6. The relationship between DOA estimation and the signal incident angle difference:

The sixth simulation shows how two signals are recognized by the MUSIC algorithm. There are two independent narrow band signals, those two signals are not correlated, the noise is ideal Gaussian white noise, the SNR is $20dB$, the element spacing is half of the input signal wavelength, array element number is 10, the number of snapshots is 500 and the incident angle is 5° and 10° and 20° respectively. The simulation results are shown in Figure 6:

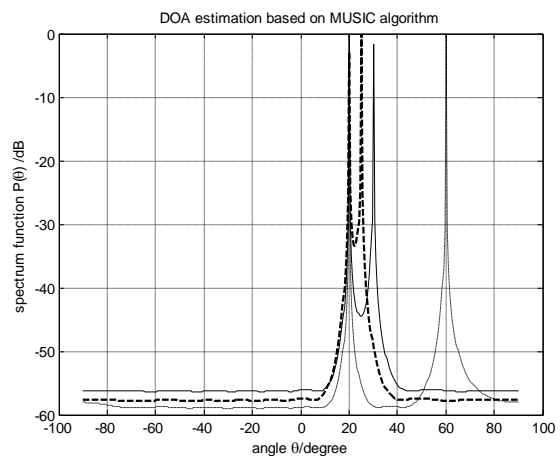


Fig. 6 simulations for the relationship between MUSIC algorithm and the incident angle difference

As can be seen from Figure 6, the dashed line is for the incident angle is 5° , the solid line for 10° and the dash-dotted line for the incident angle 20° . The other conditions remaining unchanged with the increase in incidence angle difference, the beam width of the DOA estimation spectrum becomes narrow, the direction of the signal becomes clear and the resolution of MUSIC algorithm increased. When the signal wave angle space is very small, the algorithm cannot estimate the number of signal sources.

7. The MUSIC algorithm and improved MUSIC algorithm for coherent signals:

The seventh and eighth simulations show how two signals are recognized by the MUSIC algorithm and improved MUSIC algorithm when the signals are coherent. The simulation results are shown in Figure 7 and Figure 8. As can be seen from the figures, for coherent signals, classic MUSIC algorithm has lost effectiveness, while improved MUSIC algorithm can be better applied to remove the signal correlation feature, and estimate the angle of arrival more accurately. This improved MUSIC algorithm can make DOA estimation more complete, and have a marked effect both on theoretical and practical study.

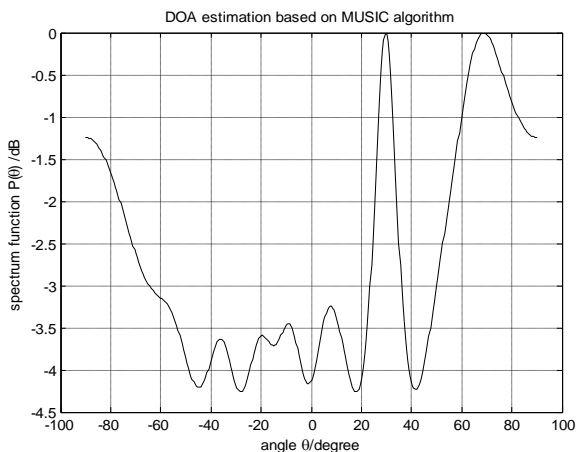


Fig. 7 Simulation for MUSIC algorithm when the signals are coherent

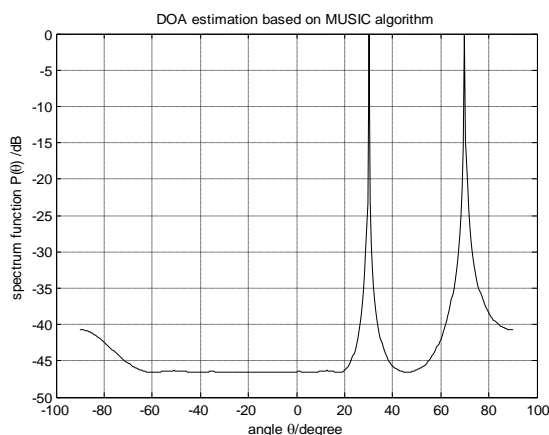


Fig.8 Simulation for the improved MUSIC algorithm when the signals are coherent

V. DISCUSSION

Estimation of DOA, based on subspace MUSIC algorithm is done and simulation results are found. It is

found that as the number of array elements, number of snapshots; the difference between the incident angles increases, resolution of the MUSIC algorithm increases. It is also seen that when the array element spacing is not more than half the wavelength, the resolution of MUSIC algorithm increases correspondingly with the increase of array element spacing. However, if the array element spacing is greater than half the wavelength, the spatial spectrum causes false peaks in other directions to the direction of signal sources. In order to solve the problem of the DOA estimation of the coherent (correlated) signals, improved algorithm is used. It is found that when the signal is coherent, classical MUSIC algorithm lost effectiveness, and improved MUSIC algorithm distinguishes their DOAs effectively.

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