Coupled Lateral and Rocking Vibration of Footings with Internal Openings

C Savitha, S Chandrakaran, T M Madhavan Pillai

Abstract*— *A study on the vibration of square and circular foundation with concentric internal holes is presented in this paper. The foundations are assumed to be rigid and embedded in, isotropic and linear elastic half-space and are subjected to coupled lateral and rocking excitation. This problem is analysed using an approximate method. The results are presented in frequency domain. Effects of embedment, mass ratio, ratio of inner to outer diameter (circular footing) or inner to outer width of the footing (square footing) and backfill are conducted to assess the behaviour. The accuracy and efficiency of the model are assessed on the basis of comparison studies with published literature. The results show that the embedment substantially affects the response in that it reduces the peak amplitudes and increases the corresponding frequencies.

Index Terms*— *Embedment, Footing, Lateral vibration, Rocking vibration.

I. INTRODUCTION

In many cases, special structures such as cooling towers, radar stations, chimneys, liquid storage tanks, etc. are built on ring, rectangular or square foundations with internal openings rather than solid foundations. The dynamic characteristics of such foundations are significantly influenced by the mere presence and the geometry of these openings. For this reason a detailed soil-foundation interaction analysis is necessary in order to assess the force-displacement relationship at the foundation level which forms the basis for a complete dynamic analysis of the superstructure. Also the prediction of coupled response to horizontal force is major importance for the design of footings exposed to dynamic effects. The methods of analysis most commonly used to determine the response characteristics are based on the assumption that the foundation rests on the surface of an elastic half- space, and the effect of embedment and side adhesion are not taken into account. However, because the actual foundations are invariably embedded, it is important to evaluate the influence of embedment on the frequency and amplitude of such foundation-soil system.

The studies on the dynamic behaviour of foundations with internal openings are rather few. Wong and Luco [1] was the first who reported study on the dynamic soil-structure interaction problem for a square foundation with internal opening. They considered rocking and vertical vibration and the analysis were based on boundary integral method. Tassoulas and Kausel [2] used the finite element method. Using consistent transmitting boundaries and hyperelements, they succeeded to obtain semi-discrete solutions for circular ring footings on layered stratum. On the basis of this methodology, they presented parametric studies on the static and dynamic stiffnesses of rigid and massless ring foundations on an elastic stratum. Veletsos and Tang [3] -[5] used spring-dashpot model to study the steady state vertical and rocking vibrations of rigid and massless ring foundations resting on homogeneous elastic half-space. Kim, et al. [6] investigated the through-the-soil steady-state interaction of a system of foundations consisting of concentric circular and ring foundations resting on elastic stratum using finite element method. In addition, Karabalis and Huang [7] studied the dynamic interaction problem of ring and square foundation with concentric opening using boundary element method. The results reported in these literatures reveal that a more comprehensive basis of results is needed for guidance in the design of structures resting on foundations with openings.

A study on the coupled horizontal and rocking vibration is presented in this paper based on an approximate theory. The foundations are assumed to be rigid. Both square and circular foundations with internal openings are considered for the study.

II. PROBLEM FORMULATION

A. Equation of Motions

The approximate theory used in the study is based on the assumption that the dynamic reactions in the footing base are equal to those of elastic half-space and that the reactions acting on the footing sides are equal to those of an overlying independent elastic layer. The equations of motion for a footing in coupled horizontal translation *u(t)* and rotation *ψ(t)* about a horizontal axis passing through the centre of gravity, based on the notations given in Fig. 1, are

$$\begin{aligned} m\ddot{u}(t) &= Q(t) - R_x(t) - N_x(t) \\ I\ddot{\psi}(t) &= M(t) - R_\psi(t) - N_\psi(t) \end{aligned} \qquad (1)$$

.in which *m*= total mass of the footing, *I*= moment of inertia about horizontal axis, passing through the centre of gravity(*C.G*), $R_x(t)$= horizontal reaction at the footing base $N_x(t)$ = resultant horizontal reaction acting on embedded surfaces (sides) of the footing, $R_\psi(t)$ = reactive moment of forces acting at footing base about centre of gravity, $N_\psi(t)$ = reactive moment of forces acting on footing sides about centre of gravity and *t* = time. The dots represent differentiation with respect to time. *Q(t)* = horizontal exciting force acting at height z_e from the centre of gravity and $M_e(t)$ = exciting moment.

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C Savitha, Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India.

Dr. S Chandrakaran, Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India.

Dr. T M Madhavan Pillai, Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India.

H is the depth of footing, l is the embedment depth of footing and z_c is the height of centre of gravity from the base of footing. Then the total moment of excitation is,

$$M(t) = Q(t)z_e + M_e(t) \tag{2}$$

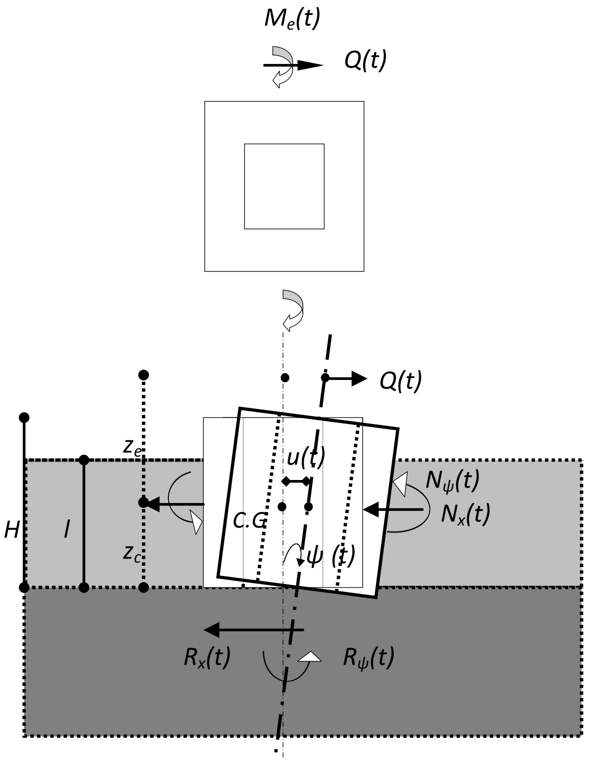

Fig. 1. Mathematical Model of the Footing

The reactive force and moment are found out based on following assumptions:

(1) The footing is of cylindrical shape.
(2) The side reactions are produced by an independent layer lying above the base of the footing.

Based on these assumptions the soil reactions are given by

$$\begin{aligned} R_x(t) &= G(r_o - r_i)(C_{u1} + iC_{u2})[u(t) - z_c\psi(t)] \\ R_\psi(t) &= G(r_o - r_i)^3(C_{\psi 1} + iC_{\psi 2})\psi(t) - G(r_o - r_i)(C_{u1} + iC_{u2})[u(t)z_c - z_c{}^2\psi(t)] \end{aligned} \tag{3}$$

Where r_0 and r_i are outer and inner radii of circular footing and

$$r_0 = \frac{2a}{\sqrt{\pi}} \quad \text{and} \quad r_i = \frac{2d}{\sqrt{\pi}} \tag{4}$$

Where 'a' and 'd' are half of outer and inner widths of the square footing as shown in Fig 2. G = shear modulusand $i = \sqrt{-1}$. If f_{ij}=flexibility coefficient or displacement functions of dimensionless frequency $a_0 = \omega r_0 \sqrt{\rho / G}$ and Poisson's ratio υ, where ω= frequency of excitation and ρ=mass density of the medium below the base, then

$$\begin{aligned} C_{u1} &= \frac{-f_{u1}}{f_{u1}{}^2 + f_{u2}{}^2} \\ C_{u2} &= \frac{f_{u2}}{f_{u1}{}^2 + f_{u2}{}^2} \\ C_{\psi 1} &= \frac{-f_{\psi 1}}{f_{\psi 1}{}^2 + f_{\psi 2}{}^2} \\ C_{\psi 2} &= \frac{f_{\psi 2}}{f_{\psi 1}{}^2 + f_{\psi 2}{}^2} \end{aligned} \tag{5}$$

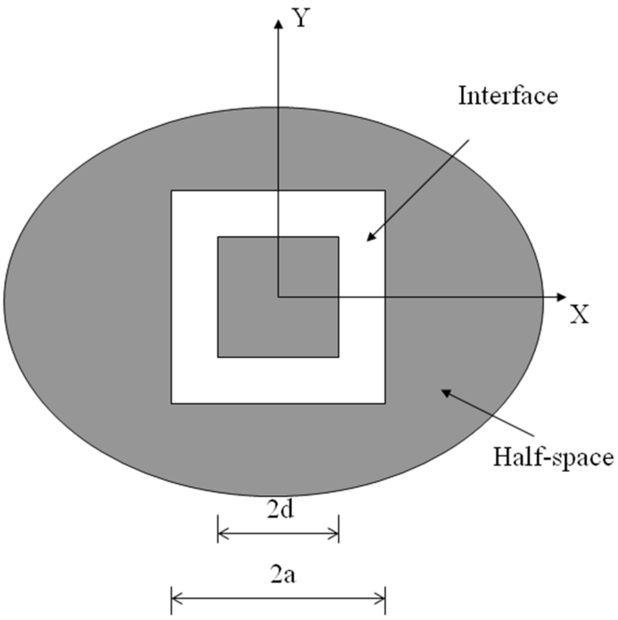

Fig. 2. Plan of the Square Footing with Concentric Internal Opening

B. Flexibility Coefficients

The amplitudes of steady state horizontal displacement, u and rotation, ψ, may conveniently be stated in the form,

$$\begin{Bmatrix} u \\ \psi r_0 \end{Bmatrix} = \begin{bmatrix} f_{u1} & f_{u2} \\ f_{\psi 1} & f_{\psi 2} \end{bmatrix} \begin{Bmatrix} \dfrac{Q}{K_x} \\ \dfrac{Mr}{K_\psi} \end{Bmatrix} \tag{6}$$

Where K_x and K_ψ are static spring coefficients. Also applying the relaxed boundary conditions at the footing-soil interface by assuming (a) zero normal contact pressure when considering the effect of a horizontal force, and (b) zero shear components of interface pressure when considering the effect of an overturning moment, coupled displacements (the rotation ψ_Q, computed for a horizontal force Q, and the horizontal displacement u_M, computed for an overturning moment M could be determined approximately. These relaxed boundary condition assumptions have the effect of partially decoupling the motions. Then using Betti's reciprocal theorem,

$$Qu_M = M\psi_Q \tag{7}$$

By combining "(6)" and "(7)", a relation interrelating the offdiagonal flexibility coefficients can be written as follows:

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$$f_{\psi 1} = r_0{}^2 \frac{K_x}{K_\psi} f_{u2} \quad (8)$$

It is clear that $f_{\psi 1}$ does not necessarily equal to f_{u2} and the relationship between these two is a function of Poisson's ratio. The expression for K_x and K_ψ for embedded footings are given by Beredugo and Novak [8]

$$K_x = \frac{9.2Ga}{2-\upsilon}\left[1+\delta^{0.8}\right]$$
$$K_\psi = \frac{4Ga^3}{1-\upsilon}\left[1+\delta+1.19\delta^2\right] \quad (9)$$

for square footings and

$$K_x = \frac{8Gr_0}{2-\upsilon}(1+\delta)$$
$$K_\psi = \frac{8Gr_0{}^3}{3(1-\upsilon)}\left[1+2.3\delta+0.58\delta^3\right] \quad (10)$$

for circular footings

Where δ is the embedment ratio $=\frac{l}{a}$ for square footing and $=\frac{l}{r_0}$ for circular footing. Horizontal and moment side reactions for circular footing can be written as:

$$N_x(t) = G_s(r_0-r_i)\delta(S_{u1}+iS_{u2})\left[u(t)+\left(\frac{l}{2}-z_c\right)\psi(t)\right]$$
$$N\psi(t) = G_s r_0 (r_0-r_i)^2\delta\left\{\left[\left(S_{\psi 1}+iS_{\psi 2}\right)+\left(\frac{\delta^2}{3}-\delta\frac{z_c}{r_0}+\frac{z_c{}^2}{r_0^2}\right)(S_{u1}+iS_{u2})\right]\psi(t) + \frac{1}{r_0}\left(\frac{\delta}{2}-\frac{z_c}{r_0}\right)(S_{u1}+iS_{u2})u(t)\right\} \quad (11)$$

Where $r_0 = \frac{2a}{\sqrt{\pi}}$ *and* $r_i = \frac{2d}{\sqrt{\pi}}$ for square footing and G_s is the shear modulus of the overlying layer.

Functions S_ψ are independent of υ and are

$$S_{\psi 1} = \pi\left[1-a_0\frac{J_0(a_0)J_1(a_0)+Y_0(a_0)Y_1(a_0)}{J_1^2(a_0)+Y_1^2(a_0)}\right]$$
$$S_{\psi 2} = \frac{2}{J_1^2(a_0)+Y_1^2(a_0)} \quad (12)$$

Here $J_n(a_0)$ *and* $Y_n(a_0)$ are Bessel functions of the first and second kinds, respectively of order '*n*'.

Functions S_u depend on Poisson's ratio and are

$$S_u(a_0,\upsilon) = G_s\left[S_{u1}(a_0,\upsilon)+iS_{u2}(a_0,\upsilon)\right]$$
$$= 2\pi G_s a_0 \frac{\frac{1}{\sqrt{q}}H_2^{(2)}(a_0)H_1^{(2)}(x_0)+H_1^{(2)}(x_0)H_1^{(2)}(a_0)}{H_0^{(2)}(a_0)H_2^{(2)}(x_0)+H_0^{(2)}(x_0)H_2^{(2)}(a_0)} \quad (13)$$

Here $q = \frac{1-2\upsilon}{2(1-\upsilon)}$, $x_0 = a_0\sqrt{q}$ and $H_n^{(2)}$ = Hankel functions of the second kind of order '*n*'.

Substitution of "(3)" and "(11)" into "(1)" yields the differential equations of coupled vibration of embedded footings:

$$m\ddot{u}(t)+(r_0-r_i)\left[G(C_{u1}+iC_{u2})+G_s\delta(S_{u1}+iS_{u2})\right]u(t)$$
$$+(r_0-r_i)\left[-Gz_c(C_{u1}+iC_{u2})+G_s\delta\left(\frac{l}{2}-z_c\right)(S_{u1}+iS_{u2})\right]\psi(t) = Q(t)$$
$$I\ddot{\psi}(t)+(r_0-r_i)^2\left[G_s\delta\left(\frac{l}{2}\delta-\frac{z_c}{r_0}\right)(S_{u1}+iS_{u2})-G\frac{z_c}{(r_0-r_i)}(C_{u1}+iC_{u2})\right]u(t) \quad (14)$$
$$+(r_0-r_i)^3\left\{G\left(C_{\psi 1}+iC_{\psi 2}\right)+\frac{G_s\delta r_0}{(r_0-r_i)}\left[\left(S_{\psi 1}+iS_{\psi 2}\right)+\left(\frac{\delta^2}{3}-\delta\frac{z_c}{r_0}+\frac{z_c^2}{r_0^2}\right)(S_{u1}+iS_{u2})\right]\right.$$
$$\left.+G\frac{z_c^2}{(r_0-r_i)^2}(C_{u1}+iC_{u2})\right\}\psi(t) = M(t)$$

With complex excitation

$$Q(t) = Q_0 e^{i\omega t}$$
$$M(t) = M_0 e^{i\omega t} \quad (15)$$

in which Q_0 and M_0 are real excitation amplitudes, the particular solutions describing the steady state motions are:

$$u(t) = u_c e^{i\omega t}$$
$$\psi(t) = \psi_c e^{i\omega t} \quad (16)$$

Where u_c and ψ_c are complex displacement amplitudes. Substitution of "(16)" into "(14)" following equations for the complex amplitudes for the coupled motion can be obtained.

$$\left[\left(k_{xx}-m\omega^2\right)+i\omega c_{xx}\right]u_c+\left(i\omega c_{x\psi}+k_{x\psi}\right)\psi_c = Q_0$$
$$\left[\left(k_{\psi\psi}-I\omega^2\right)+i\omega c_{\psi\psi}\right]\psi_c+\left(i\omega c_{x\psi}+k_{x\psi}\right)u_c = M_0 \quad (17)$$

This is the formal equation for a two-degree of freedom system in which frequency dependent stiffness constants are:

$$k_{xx} = G(r_0-r_i)\left(C_{u1}+\frac{G_s}{G}\delta S_{u1}\right)$$
$$k_{\psi\psi} = G(r_0-r_i)^3\left[C_{\psi 1}+\left(\frac{z_c}{(r_0-r_i)}\right)^2 C_{u1}+\frac{G_s r_o}{G(r_0-r_i)}\delta S_{\psi 1}+\frac{G_s r_o}{G(r_0-r_i)}\delta\left(\frac{\delta^2}{3}+\frac{z_c^2}{r_0^2}-\delta\frac{z_c}{r_0}\right)S_{u1}\right] \quad (18)$$
$$k_{x\psi} = -G(r_0-r_i)\left[z_c C_{u1}+\frac{G_s}{G}\delta\left(z_c-\frac{l}{2}\right)S_{u1}\right]$$

and frequency dependent damping constants are:

$$c_{xx} = \frac{G(r_0-r_i)}{\omega}\left(C_{u2}+\frac{Gs}{G}\delta S_{u2}\right)$$
$$c_{\psi\psi} = \frac{G(r_0-r_i)^3}{\omega}\left[C_{\psi 2}+\left(\frac{z_c}{(r_0-r_i)}\right)^2 C_{u2}+\frac{G_s r_0}{G(r_0-r_i)}\delta S_{\psi 2}+\frac{G_s r_0}{G(r_0-r_i)}\delta\left(\frac{\delta^2}{3}+\frac{z_c^2}{r_0^2}-\delta\frac{z_c}{r_0}\right)S_{u2}\right] \quad (19)$$
$$c_{x\psi} = -\frac{G(r_0-r_i)}{\omega}\left[z_c C_{u2}+\frac{G_s}{G}\delta\left(z_c-\frac{l}{2}\right)S_{u2}\right]$$

C. *Calculation of Vibration Amplitudes*

From "(17)" the complex vibration amplitudes can be written as

$$u_c = Q_0\frac{\alpha_1+i\alpha_2}{\varepsilon_1+i\varepsilon_2}$$
$$\psi_c = M_0\frac{\beta_1+i\beta_2}{\varepsilon_1+i\varepsilon_2} \quad (20)$$

Where

$$\alpha_1 = k_{\psi\psi} - I\omega^2 - \frac{M_0}{Q_0} k_{x\psi}$$
$$\alpha_2 = \left(c_{\psi\psi} - \frac{M_0}{Q_0} c_{x\psi} \right)\omega$$
$$\beta_1 = k_{xx} - m\omega^2 - \frac{M_0}{Q_0} k_{x\psi} \qquad (21)$$
$$\beta_2 = \left(c_{xx} - \frac{Q_0}{M_0} c_{x\psi} \right)\omega$$
$$\varepsilon_1 = mI\omega^4 - \left(mk_{\psi\psi} + Ik_{xx} + c_{xx}c_{\psi\psi} - c_{x\psi}^2 \right)\omega^2 + \left(k_{xx}k_{\psi\psi} - k_{x\psi}{}^2 \right)$$
$$\varepsilon_2 = -\left(mc_{\psi\psi} + Ic_{xx} \right)\omega^3 + \left(c_{xx}k_{\psi\psi} + c_{\psi\psi}k_{xx} - 2c_{x\psi}k_{x\psi} \right)\omega$$

Also

$$u_c = u_1 + iu_2$$
$$\psi_c = \psi_1 + i\psi_2$$
$$and \qquad (22)$$
$$u_0 = \sqrt{u_1^2 + u_2^2} = Q_0 \sqrt{\frac{\alpha_1^2 + \alpha_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$$
$$\psi_0 = \sqrt{\psi_1^2 + \psi_2^2} = M_0 \sqrt{\frac{\beta_1^2 + \beta_2^2}{\varepsilon_1^2 + \varepsilon_2^2}}$$

Where u_0 and ψ_0 are real vibration amplitudes.

As in the case of uncoupled modes, dimensionless amplitudes are $A_x = u_0 G r_0 / Q_0$ and $A_\psi = \psi_0 G r_0^2 / M_0$ may be introduced to facilitate the presentation and analysis of the results.

Frequency variable excitation, often encountered in practical cases, can be easily introduced into the above formulae. Assuming a frequency variable horizontal excitation, caused by an unbalanced rotating mass 'm_e', acting at a height z_e above the centre of gravity, then in "(20)" to "(22)"

$$Q_0 = m_e e\omega^2, \qquad M_0 = m_e e\omega^2 z_e, \qquad \frac{M_0}{Q_0} = z_e \qquad (23)$$

in which 'e' is the rotating mass eccentricity. The dimensionless vibration amplitudes are $A_x = u_0 m / m_e e$ and $A_\psi = \psi_0 I / m_e e z_e$.

D. Natural Frequencies and Modes

In addition to the computation of the complete response, the natural undamped frequencies and modes of free vibration can be of interest and are useful in the direct resonant amplitude calculation.

The equations for the natural frequencies and modes can be obtained by putting damping coefficients as well as excitation forces equal to zero, which yields, in terms of real amplitudes,

$$\begin{bmatrix} k_{xx} - m\omega^2 & k_{x\psi} \\ k_{x\psi} & k_{\psi\psi} - I\omega^2 \end{bmatrix} \begin{Bmatrix} u_0 \\ \psi_0 \end{Bmatrix} = 0 \qquad (24)$$

Then from the condition that determinant of the coefficients is equal to zero,

$$\omega_{1,2}^2 = \frac{1}{2}\left(\frac{k_{xx}}{m} + \frac{k_{\psi\psi}}{I} \right) \pm \sqrt{\frac{1}{4}\left(\frac{k_{xx}}{m} - \frac{k_{\psi\psi}}{I} \right)^2 + \frac{k_{x\psi}{}^2}{\mathrm{Im}}} \qquad (25)$$

With these two natural frequencies the two vibration modes (eigenvectors) are

$$\left(\frac{u_0}{\psi_0} \right)_j = \frac{-k_{x\psi}(\omega_j)}{k_{xx}(w_j) - m\omega_j^2} = \frac{k_{\psi\psi}(\omega_j) - I\omega_j^2}{-k_{x\psi}(\omega_j)}$$
$$j = 1,2 \qquad (26)$$

The first mode represents the rotation about a centre lying below the base of the footing and second mode represents the rotation about a point lying above the centre of gravity.

III. EXAMPLES OF THEORETICAL RESPONSE CURVES

Several examples of theoretical response curves computed from "(22)" are plotted. In these figures the horizontal translation and rocking components are plotted for various relative embedment's. Two sets of mass parameters are used to indicate their effects on the character of the response. The modified mass ratios are $B_x = b_x(7-8\upsilon)/32(1-\upsilon)$ and $B_\psi = 3b_\psi(1-\upsilon)/8$, where mass ratios $b_x = m/\rho r_0^3$ and $b_\psi = I/\rho r_0^5$. Fig. 3 shows the comparison of the coupled horizontal translation for a solid circular footing with the published one [8]. It can be seen that the result matches very well.

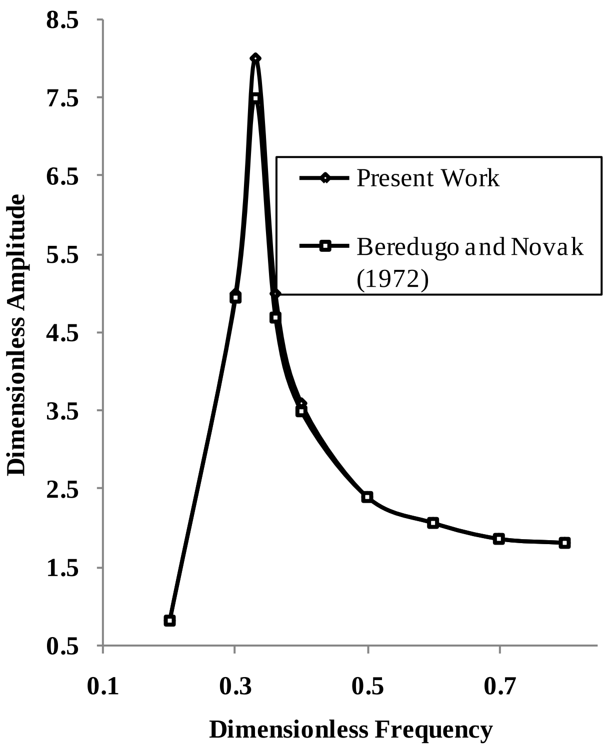

Fig.3. Theoretical response curves for coupled horizontal motion of circular footing (B_x=4, B_ψ=4.35, υ=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, l/r_o =0.5, and r_i/r_o=0)-Comparison of the results

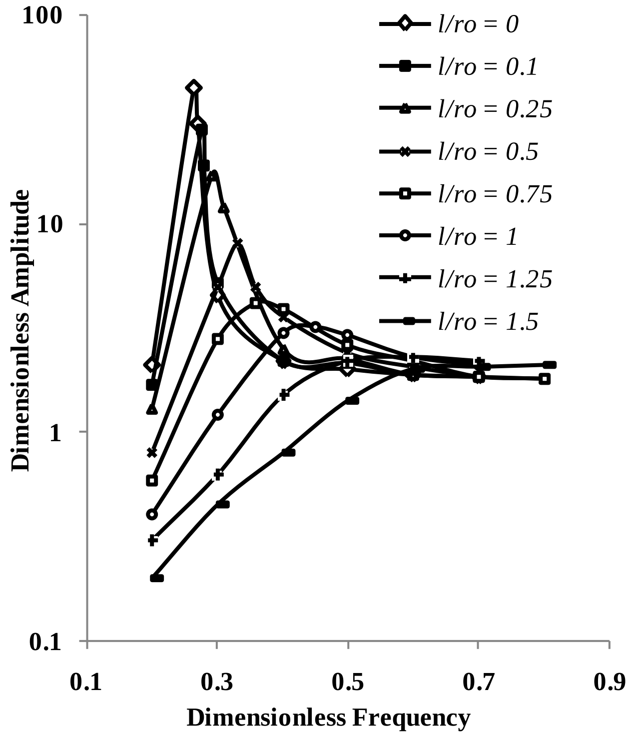

Fig.4. Theoretical Response Curves for Coupled Horizontal Motion of Circular Footing with Various Embedment Ratio (B_x=4, B_ψ=4.35, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, and r_i/r_o=0)

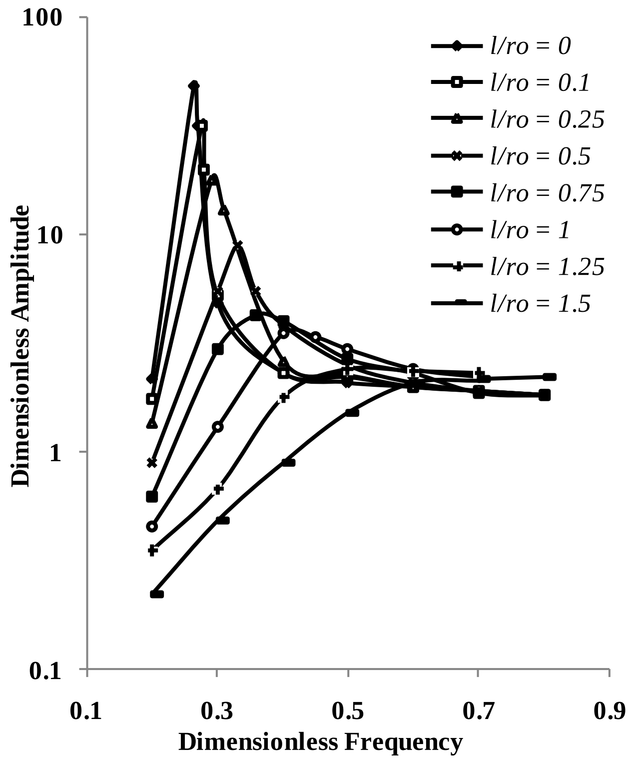

Fig.5. Theoretical Response Curves for Coupled Horizontal Motion of Circular Footing with Various Embedment Ratio (B_x=4, B_ψ=4.35, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, and r_i/r_o=0.25)

Figs. 4 and 5 show the coupled horizontal translation of circular footings with internal openings with various embedments. It can be seen that the embedment substantially affects the response in that it increases the frequencies and reduces the amplitudes in both the cases. Moreover the response in all cases is usually dominated by the first peak amplitude and the second peak is entirely suppressed. Hence the due importance should be given to first peak.

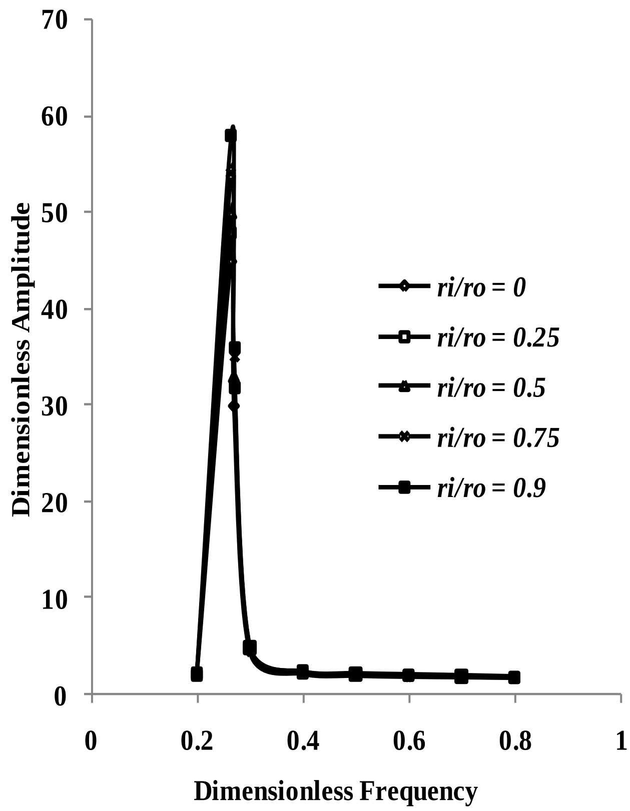

Fig.6. Theoretical Response Curves for Coupled Horizontal Motion of Circular Footing with Various r_i/r_o Ratio (B_x=4, B_ψ=4.35, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, and l/r_o=0)

Fig. 6 shows the variation of the response with respect to r_i/r_o ratios for a surface footing. Here for all ratios the behaviour is similar. That is footings with all ratios have a unique peak. The rocking components of circular footings for various relative embedments are depicted in Fig. 7. and that for various r_i/r_o ratios are depicted in Fig. 8. Here also the behaviour is similar to that of coupled horizontal translation. But second peak is distinct in these cases.

Fig. 9 shows the effect of embedment on the coupled horizontal translation of square footing with *d/a*= 0.5. The increase in the first peak frequency and the decrease in the corresponding amplitude due to embedment are quite drastic in this case. The variation of response with different d/a ratios for an embedment ratio *l/a*= 0.5 is shown in the Figs. 10 and 11 for different mass ratio. Here the footing with *d/a*= 0.9 shows higher peak in both the cases. All the footings have the first peak at the same frequency. Fig. 12 shows the effect of embedment on the behaviour of the square footing with *d/a* =0.5 for the coupled rocking motion. The behaviour is similar to that of circular footing with internal openings. All these figures illustrate the steady state response of footings embedded in undisturbed soil.

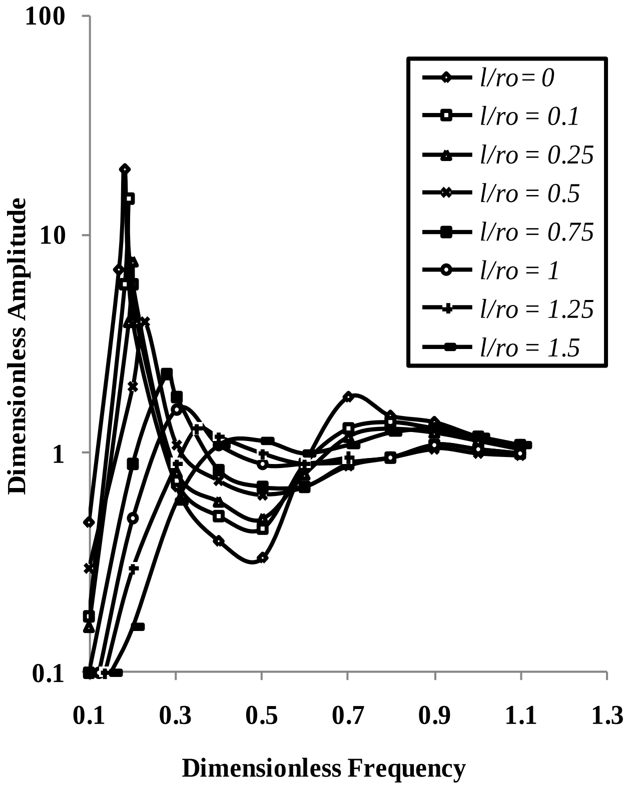

Fig.7. Theoretical Response Curves for Rocking Component in Coupled Motion of Circular Footing with Various Embedment Ratio (B_x=8, B_ψ=8.7, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, and r_i/r_o=0.25)

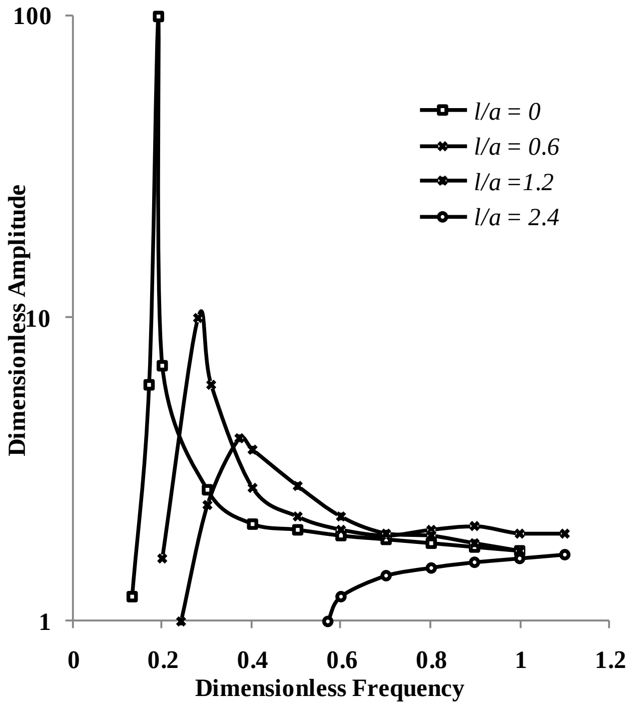

Fig.9. Theoretical Response Curves for Horizontal Translation in Coupled Motion of Square Footing with Various Embedment Ratio (B_x=3, B_ψ=6.4, ν=0, z_c/a = 1.7, z_e/a=1.9, H/a =3.17, and d/a=0.5)

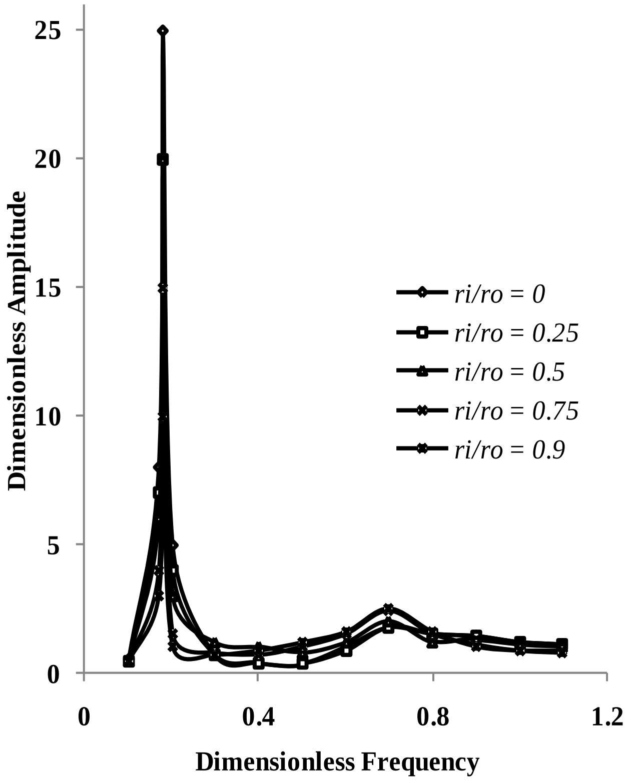

Fig.8. Theoretical Response Curves for Rocking Component in Coupled Motion of Circular Footing with Various r_i/r_o Ratio (B_x=8, B_ψ=8.7, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, and l/r_o=0)

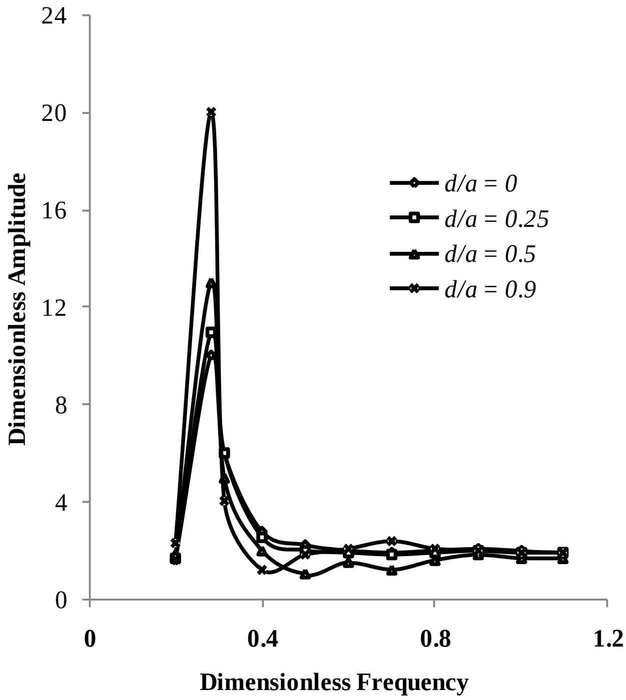

Fig.10. Theoretical Response Curves for Horizontal Translation in Coupled Motion of Square Footing with Various d/a Ratio (B_x=3, B_ψ=6.4, ν=0, z_c/a = 1.7, z_e/a=1.9, H/a =3.17, and l/a=0.5)

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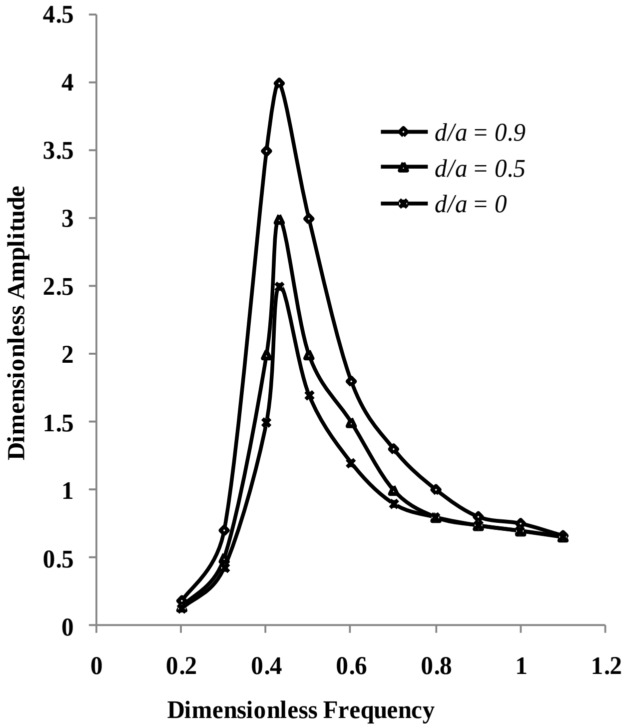

Fig.11. Theoretical Response Curves for Rocking Component in Coupled Motion of Square Footing with Various *d/a* Ratio (B_x=1, B_ψ=2.13, ν=0, z_c/a = 1.7, z_e/a=1.9, *H/a* =3.17, and *l/a*=0.5)

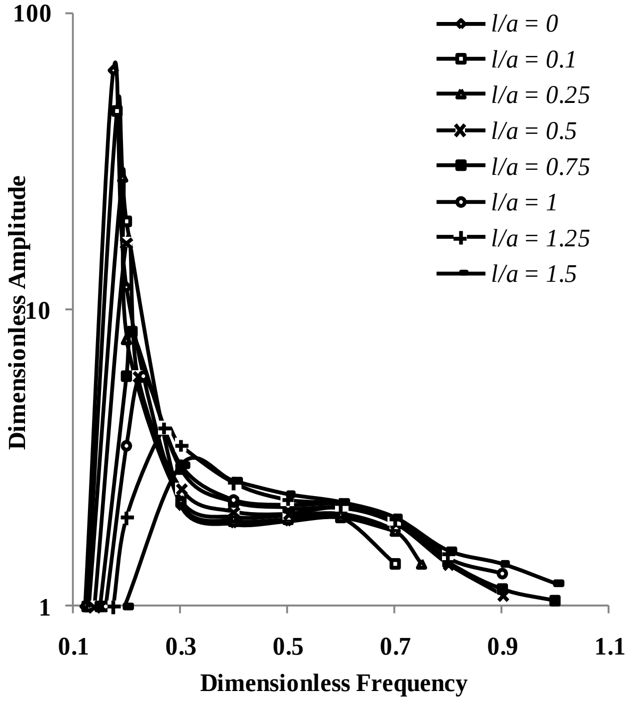

Fig.13. Theoretical Response Curves for Horizontal Translation in Coupled Motion of Circular Footing with Various Embedment Ratio (B_x=8, B_ψ=8.7, ν=0, z_c/r_o = 1.08, z_e/r_o=1.12, H/r_o =2, η=0.75, and r_i/r_o =0.5)

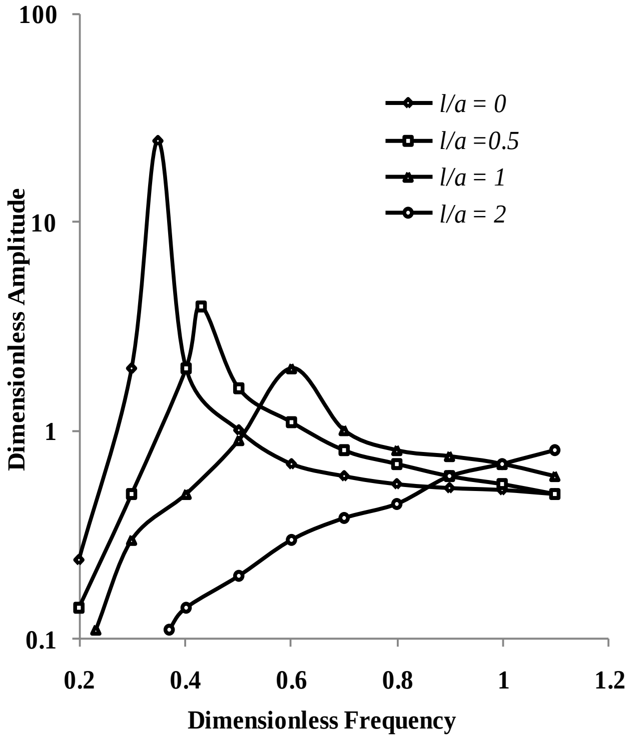

Fig.12. Theoretical Response Curves for Rocking Component in Coupled Motion of Square Footing with Various Embedment Ratio (B_x=1, B_ψ=2.13, ν=0, z_c/a = 1.7, z_e/a=1.9, *H/a* =3.17, and d/*a*=0.5)

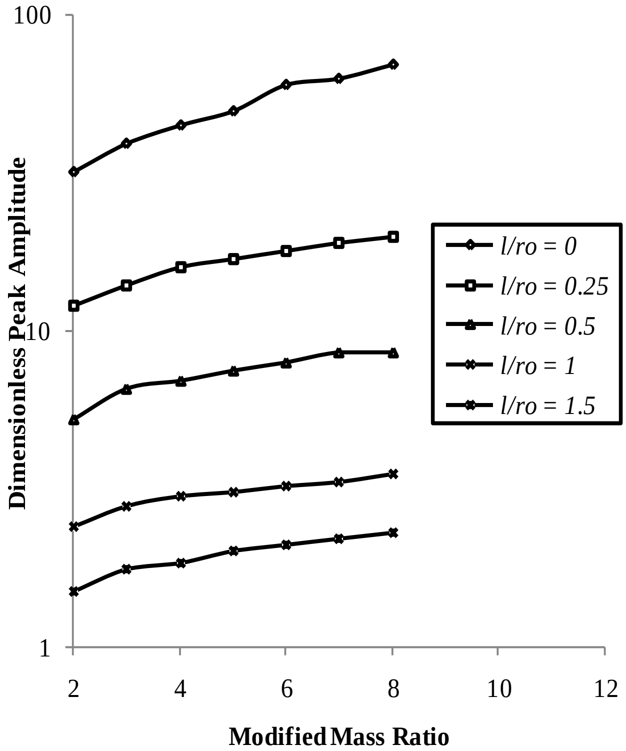

Fig. 14. Effect of Modified Mass Ratio on Theoretical Resonant Amplitude for Horizontal Translation (Circular Footing)

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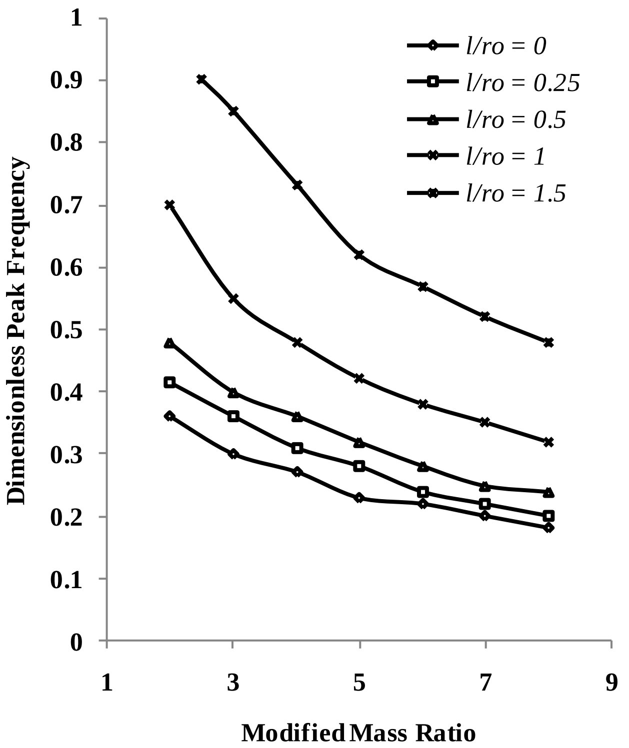

Fig. 15. Effect of Modified Mass Ratio on Theoretical Resonant Frequency for Horizontal Translation (Circular Footing)

Fig. 13 illustrates the effect of backfill. The properties of backfill G_s, ρ_s were introduced using an approximate expression $G_s/G \approx (\rho_s/\rho)^3$, Thus the ratio $\eta = \rho_s/\rho$ =1 denotes embedment in undisturbed soil. In all the figures the response of embedded footings is dominated by the first peak, which is thus of major importance. The second peak is, in general, much less pronounced and does not vary too much with embedment. In most practical cases, the second peak is entirely suppressed; it can be recognized in the rocking components with very high mass ratios. Both of these effects are smaller in the case of backfill.

The variations of the peak amplitudes and peak frequencies with relative embedment and mass ratios are further illustrated in Figs.14 and 15 respectively. It can be seen that with increase in the mass ratio there is slight increase in the peak amplitude and decrease in the corresponding frequency for all embedment ratios.

These figures apply exactly just for the parameters used in the computing; however, they indicate the trends to be expected in any particular case.

IV. SUMMARY AND CONCLUSIONS

Coupled vibration in lateral and rocking of square and circular footings with internal openings that are partially embedded was investigated theoretically. An approximate analytical solution was used to derive directly usable formulas and information about embedment into backfill. The major findings can be summarized as follows:

(1) The response in all cases is usually dominated by the first peak and the second peak is entirely suppressed.

(2) The embedment substantially affects the response in that it reduces the peak amplitudes increases the corresponding frequencies.

(3) Backfill reduces the effect of embedment.

(4) For square footings the effect of d/a ratio on the response is negligible.

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AUTHORS PROFILE

C Savitha, is a Research Scholar in Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India. She did her B Tech in Civil Engineering at Government Engineering College Kannur, Kerala and M Tech in Geotechnical Engineering at College of Engineering Trivandrum, Kerala. This paper is a part of research work done at National Institute of Technology, Calicut, Kerala under the guidance of Dr. S. Chandrakaran, the Second Author and Dr T M Madhavan Pillai, the Third Author. This study is supported by scholarship from the Ministry of Human Resourse Development, Government of India.

Dr. S Chandrakaran is Professor in Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India. He held various positions in the Institute and now working as Dean, Planningand Development. He has published a number of papers in the field of Geotechnical Engineering. He has guided a number of students in M Tech and Ph D Programme.

Dr. T M Madahavan Pillai is Professor in Department of Civil Engineering, National Institute of Technology, Calicut, Kerala, India. His area of interest is in Structural Engineering. He has published a number of papers in the field of Structural Engineering. He has guided a number of students in M Tech and Ph D Programme.

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