Optimization and Empirical Analysis of Portfolio Model

Rui Zhang, Fang Chen

Abstract: ***Because the investor's subjective risk preference and the choice of the parameter is different, this paper makes a mathematical modeling for the multi objective portfolio model and transforms it into a single target model. On the other hand, the parameter function is transformed into a linear programming problem, and the optimal investment combination scheme is obtained. Investors can directly choose their own investment direction and make an empirical analysis based on two opposing goals, which are as large as possible and risk as small as possible.***

Keywords: Optimization of Portfolio Model, Mathematical Modeling, Linear Programming, Empirical Analysis.

I. INTRODUCTION

In the investment of the risk market, the consumer's investment preference not only hopes to get as much income as possible, but also wants to take as small as possible risks. These two problems are difficult to deal with at the same time. So how to determine the risk and income of portfolio investment and how to balance the two indexes for asset allocation is an urgent need to be solved for the market investors. Therefore, this paper adopts the analysis of several representative stocks in Shanghai and Shenzhen 300. Through the mathematical optimization and linear optimization of the traditional Markowitz mean variance portfolio model, Excel and MATLAB software are used to calculate the optimized model, and the maximum income and the minimum risk of the optimal scheme are made respectively.

II. OPTIMIZATION OF THE MODEL BASED ON MEAN VARIANCE MODEL

2.1. The Establishment of the Model

According to the contents of the securities investment science, the expected return rate of portfolio investment is the weighted average of the expected return rate of various securities in the portfolio. However, the risk of portfolio investment is not the weighted average of each standard deviation. The portfolio theory shows that the risk of portfolio investment is generally less than the risk of a single investment, so in addition to the individual risks that constitute the securities, the degree of correlation should be considered, namely, the introduction of two mathematical variables: the standard deviation and the correlation coefficient. Therefore, the mean variance model is mathematically optimized, and the multi-objective function is transformed into a single objective function. The model is as follows:

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$$\sigma_p = \sqrt{\sigma_1^2 P_1^2 + \sigma_2^2 P_2^2 + \sigma_3^2 P_3^2 + 2P_1P_2r_{12}\sigma_1\sigma_2 + 2P_1P_3r_{13}\sigma_1\sigma_3 + 2P_2P_3r_{23}\sigma_2\sigma_3}$$

$$\overline{Y_P} = \sum_{i=1}^{n} \overline{y_i} \cdot P_i$$

2.2. A brief Description of Symbols

The σ_p indicate Portfolio risk, σ_1 indicate the risk of securities 1, p_1 indicate the weight of securities 1, γ_{12} indicate the correlation coefficient of securities 1 and securities 2, $\overline{Y_P}$ indicate expected rate of return on portfolio investment, $\overline{Y_i}$ indicate expected rate of return of i securities, P_i indicate the weights.

2.3. Model Solution

(1) Input data, calculate the risk of securities according to the rate of return (using the rise and fall rate to measure the yield of securities), and use the STDEV.P (Number1, number2,···) function in the Excel software to calculate the standard deviation (using the standard deviation to measure the risk);

(2) According to the rate of return, the correlation coefficient between 22 securities is calculated, and the CORREL function is applied;

(3) The weight is allocated reasonably, and the optimization model is calculated by using Excel software;

(4) According to the calculation results, we analyze the optimal portfolio model.

2.4. Empirical Analysis of Model Based on Excel Software

a. Research of Problem

In this paper, from the selection of the csi 300 in 2017 to three representative belong to different industries, respectively is cofco sugar zhongyuan high-speed focus media, this way of picking stocks on the one hand, weaken the relevance of the stock, reduce the systemic risk, but to a certain extent, reflects the market risk in measuring market risk, choose the Shanghai composite index and shenzhen component index on behalf of the entire market average yield trial 200000 to buy three shares, to form a portfolio, analysis how to distribute the weight of the three stocks do risk minimum (source: netease financial stocks).

b. According to the model, the correlation coefficient between the standard deviation of three securities (see 1-4-2-1)

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was calculated by using the STDEV.P function and CORREL function in the Excel software.

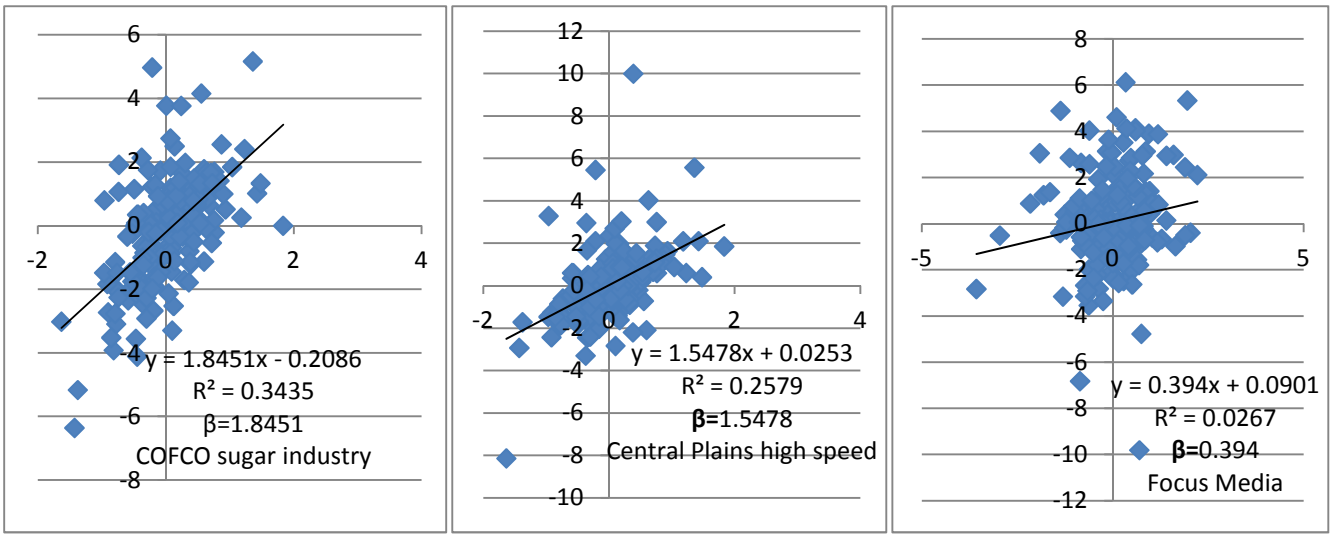
	COFCO sugar industry	Central Plains high speed	Focus Media
standard deviation	1.639778505	1.585133846	1.967264403
	COFCO sugar industry and Central Plains high speed	Central Plains high speed and Focus Media	Focus Media and COFCO sugar industry
correlation coefficient	0.438862606	0.061612976	0.020346222

c. Using the optimized portfolio model, the weights of three securities are allocated, according to the table 1, the optimal portfolio plan is as follows (see 1-4-3-1).

Weight distribution (wan)			The corresponding standard deviation, Risk value
COFCO sugar industry	Central Plains high speed	Focus Media	
7	7	6	0.0114853

d. Measurement of Systemic Risk.

In the following figure, the direction of the three stocks' change is consistent with the direction of the stock market, but the magnitude of the fluctuation is different. The size of the beta value represents the fluctuation range of all kinds of stocks with the stock market. As can be seen from the picture, the change range of the grain sugar industry and the Central Plains high speed is greater than the change range of the stock market. The fluctuation range of the Focus Media is less than the change range of the stock market, and the beta value of the grain sugar industry is the largest, the high speed beta of the Central Plains is the second and the Focus Media's beta value is the smallest. The greater the beta value of a security, the greater the systemic risk. Therefore, the systemic risk of the three groups is from large to small in the following respects: COFCO sugar industry, Zhongyuan Expressway and Focus Media.



Systemic Risk

e. Interpretation of Result

Analysis of data obtained from 1-4-3-1, the investment grain sugar 70000、zhongyuan high-speed 70000 and investment of focus media 60000 is minimum risk of the portfolio, only 1.111% of the risk, under this weight distribution is based on the results from these risks under the condition of minimum combined yield value, this model suggested that risk aversion is used but it is important to note that in order to make the result more accurate, investors maximization goal, as far as possible when we assign weights allocation groups, hundreds of thousands of group allocation can make the result close to perfect.

III. PARAMETRIC FUNCTIONS ARE TRANSFORMED INTO MODEL OPTIMIZATION FOR LINEAR PROGRAMMING PROBLEMS

3.1. Hypothesis of Model

(1) The amount of money invested by a securities firm is quite large, and there is no loan or overdraft.
(2) Investors estimate the net income of a portfolio based on the average return on the stock minus the transaction cost.
(3) Investors estimate the risk of a portfolio based on the risk of a security loss.

3.2. A Brief Description of Symbols

The S_i is the ith asset asset (i=1,2, ⋯,n,n+1), where S_{n+1} is deposited in the bank; X_i is the proportion of the total investment in S_i , hereinafter referred to as investment; Y_i is the transaction cost of investment S_i is the proportion of total amount,

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Hereinafter referred to as transaction fee; R_i is average yield on S_i; Q_i is risk loss ratio of S_i;P_i is transaction rate of S_i; M is capital investment;f_1 is net profit; f_2 is overall risk.

3.3. Selection of Model Parameters

Because in the actual transaction, investors need to consider the transaction process of transaction costs, risk of loss, therefore in the establishment of the model, the measured by income minus the transaction fee investor's net income, with the risk of securities losses to measure the risk of portfolio. In the traditional model, the transaction cost function and risk function are considered, so in the optimization model, the nonlinear function is transformed into a linear programming problem.

3.4. Model Optimization

Because the target function contains two parameters and two decision targets, F1 and Yi are not continuous functions of Xi, which means that the net income F1, the transaction cost Yi and the investment amount Xi are not a smooth line, leading to the difficulty of solving the problem. Therefore, when the model is optimized, the transaction cost function needs to be linearized, the risk function is transformed, and an infinite approach to the nonlinear solution is obtained, and the optimization problem with multiple decision targets is transformed into a single objective problem by using the weighted method. The final objective function is as follows:

$$\min f = (1-\lambda)\sum_{i=1}^{n+1}(p_i - r_i)X_i + \lambda X_{n+2}$$

$$s.t\begin{cases}\sum_{i=1}^{n+1}(1+p_i)X_i = 1 \\ q_i X_i - X_{n+2} \le 0 \ldots i = 1,2,\cdots n \\ X_i \ge 0 \ldots i = 1,2,\cdots n+2\end{cases}$$

3.5 The Empirical Analysis of the Model based on Matlab Software.

Question research: to design a portfolio,with a given money M selectively buy several kind of interest-bearing assets or deposit bank, make the net income as large as possible, and the overall risk as small as possible. The linprog function in the matlab workbox is used to solve the above linear programming problem.

The relevant data at n=4 are as follows:

S_i	R (%)	Q (%)	P (%)	u（元）
S_1	28	2.5	1	103
S_2	21	1.5	2	198
S_3	23	5.5	4.5	52
S_4	25	2.6	6.5	40

Because linprog function in MATLAB only solves linear programming problems. So when we replace the data in the above table into the total objective function, we should transform the data in the above table into the following forms.

$$\min \quad f = (-0.05, -0.27, -0.19, -0.185, -0.185)^T (x_0, x_1, x_2, x_3, x_4)$$

$$\begin{cases} x_0 = x_0 + 1.01x_1 + 1.02x_2 + 1.045x_3 + 1.065x_4 = 1 \\ 0.025x_1 \le a \\ 0.015x_2 \le a \\ 0.055x_3 \le a \\ 0.026x_4 \le a \\ x_i (i = 0,1,2,3,4) \ge 0 \end{cases}$$

Because a is any given risk degree, different investors have different risk degrees. We start from a=0, using the step length Delta a=0.003 for cyclic search. In the calculation results obtained by the LP function, the optimal portfolio is about a*=0.6%, q*=20%, and the corresponding portfolio scheme is shown as follows:

Investment proportion					Risk degree	Profit
X_0	X_1	X_2	X_3	X_4		
0.0297	0.2400	0.4000	0.1091	0.2212	0.0060	0.2019

IV. SUMMARY

In this paper, the model is optimized by mathematical optimization and parameter function optimization. The optimized model can be solved by using the existing software MATLAB and excel directly. It is free from heavy calculation and high program language debugging, and can realize our calculation process in the shortest time. In addition to considering the benefit risk, the optimized model also adds the influence of the rise and fall, the transaction cost and the net income to the model, making the optimized model closer to the real operation, and the optimal portfolio scheme is more perfect.

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