



A Novel Extended Cyclic MUSIC algorithm using Wavelet Decomposition Technique

N. V. S. V. Vijay Kumar, K. Raja Rajeswari, P. Rajesh Kumar

Abstract- The study and analysis of direction of arrival(DOA) estimation has been a very challenging aspect in wireless communication systems, radar and sonar in the presence of a fading channel. Extended cyclic MUSIC subspace based method is reviewed for the DOA when noise is present. An extension of the Cyclic MUSIC algorithm is the Extended mode of the cyclic MUSIC which uses an extended version of the data array vector. Based upon the spectral correlation of the inbound signals these algorithms can estimate the signals' direction of arrival. The spatially correlated noises, present degrade the performance of the Cyclic MUSIC algorithms and also fail to make an estimate of the signals' direction of arrival when they are closely spaced. The paper proposes an improved DOA estimation technique of Wavelet Decomposition when spatially correlated noises are present. The signals present at the receiver side are denoised and the required estimates of the transmitted signals are obtained by using the Extended Cyclic MUSIC algorithm to the denoised data. Simulations prove that the output SNR has been enhanced considerably.

Index Terms - Multiple-Input Multiple-Output(MIMO) radar, Direction of arrival, cyclostationarity, cyclic MUSIC, Extended cyclic MUSIC, Wavelet Decomposition.

I. INTRODUCTION

The very basic and most important aspect of signal processing is to regenerate the original signal under noisy conditions. High resolution techniques like Multiple signal classification (MUSIC) are used for calculating the directions of arrival of a large number of plane waves under noisy environments [1]. The performance of classical MUSIC algorithm degrades significantly if the signals are spatially correlated[2]. Also the algorithm fails to calculate the direction of arrivals of the arrays when they are closely spaced. So we analyze using a variant of MUSIC which is the Extended Cyclic MUSIC algorithm[3]. This algorithm uses the cyclic correlation matrix that bounds the cyclo-stationarity property of the received signals in lieu of the traditional correlation matrix. Due to the signal selective property[4] and the direction finding nature of the algorithm, it selects the desired signals ignoring the noisy interferences. Here the wavelet decomposition method is applied to the above algorithm to verify the performance of the method in a noisy environment. This Wavelet decomposition technique[5-7] employed enhances the signal to noise(SNR) ratio of the sensor arrays at the output. The Extended Cyclic MUSIC algorithm is then applied to the denoised data for the DOA estimation[8-9].

The effect of Wavelet decomposition on the overall performance of the Extended Cyclic MUSIC is calculated by taking the simulations of the output SNR that has not only improved invariably but has also enhanced the resolution of the closely spaced direction of arrivals. This proves that the Wavelet decomposition technique significantly improves the performance of the Extended Cyclic MUSIC algorithm.

II. DATA MODEL

A uniformly spaced linear array(ULA) is considered with P isotropic sensors that are equally spaced with distance of l are considered for this model. The beamforms of the narrow band signals from different directions arrive at different angles $\theta_1, \theta_2, \dots, \theta_m$. The sampled received data of the p^{th} sensor is given by

$$z_p(t) = \sum_{r=1}^K a_r(t) e^{j\omega t + j(p-1)\sin\theta_r} + w_p(t) \quad (1)$$

where $a_r(t)$ is the r^{th} zero mean complex random signal amplitude, q is the wave number and $w_p(t)$ the zero-mean additively white Gaussian noise (AWGN). The above equation representation in matrix form is given by

$$z = Cv + y \quad (2)$$

where $z = [z_1(t), z_2(t), \dots, z_p(t)]^T$, $v = [v_1(t)e^{j\omega t}, v_2(t)e^{j\omega t}, \dots, v_n(t)e^{j\omega t}]^T$ is the signal vector at the source and $y = [y_1(t), y_2(t), \dots, y_p(t)]^T$ is the noise signal vector. Matrix C is given by

$$C = [c(\omega_1), c(\omega_2), \dots, c(\omega_K)] \quad (3)$$

where $c(\omega_k)$ is the steering vector. Consider the signals to be zero-mean and also wide sense stationary(WSS) are uncorrelated and have the variance each of σ^2 . The covariance matrix is evaluated below

$$\begin{aligned} \mathbf{R} &= E[zz^H] \\ &= CE[ss^H]C^H + E[yy^H] \\ &= CDC^H + \sigma^2\mathbf{I} \end{aligned} \quad (4)$$

where $D = E[ss^H]$ and $E[yy^H] = \sigma^2\mathbf{I}$.

The eigen vector \mathbf{R} is given by

$$\mathbf{R} = [\mathbf{T} \ \mathbf{U}] \begin{bmatrix} \Lambda & \mathbf{0} \\ \mathbf{0} & \sigma^2\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{T}^H \\ \mathbf{U}^H \end{bmatrix} \quad (5)$$

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* Correspondence Author

N. V. S. V. Vijay Kumar, GITAM University, Visakhapatnam, India. E-mail: vijaynandam@gmail.com

K. Raja Rajeswari, G.V.P College of Engineering for Women, Visakhapatnam, India. E-mail: kkrauv@yahoo.com

P. Rajesh Kumar, Andhra University College of Engineering, Visakhapatnam, India. E-mail: rajeshauce@gmail.com

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T and **U** are the eigen vector matrices of the signal and noise subspaces.

The estimate for **R** taken from the **K** data samples is given by

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{l=1}^K z(t) z^H(t) \quad (6)$$

So with the eigen decomposition matrix $\hat{\mathbf{R}}$, we can estimate the noise subspace eigen vector matrix as $\hat{\mathbf{U}}$. From this the MUSIC spectrum is given by

$$p(\omega) = \frac{1}{c^H(\omega) \hat{\mathbf{U}} \hat{\mathbf{U}}^H c(\omega)} \quad (7)$$

Here the noise spectrum is spatially correlated so the MUSIC algorithms degrade substantially.

III. EXTENDED CYCLIC MUSIC

The Extended mode of cyclic MUSIC is an extension for the conventional cyclic MUSIC algorithm which uses the cyclostationarity property of the received signals. The extended data vector is formed by

$$d_E(t) = \sum_{m=1}^M G(\omega_m) \begin{bmatrix} v_m(\omega) \\ v_m^*(\omega) \end{bmatrix} + \begin{bmatrix} y(\omega) \\ y^*(\omega) \end{bmatrix} \quad (8)$$

where $G(\omega) = [c_1(\omega) \quad c_2(\omega)]$

Taking the values of ω , we have

$$c_1^H(\omega) c_2(\omega) = 0 \quad (9)$$

$$c_1^H(\omega) c_1(\omega) = c_2^H(\omega) c_2(\omega) = \gamma \quad (10)$$

where $\|c(\omega)\|^2 = \gamma$.

Going forward evaluation of the cyclic-correlation matrix is done on the extended-data model by

$$\mathbf{R}_E = \frac{1}{K} \sum_{k=1}^K \mathbf{I}_{2L} z_E z_E^H \quad (11)$$

where L -dimensional identity matrix \mathbf{I}_{2L} is defined by

$$\mathbf{I}_{2L} = \begin{bmatrix} \mathbf{I}_L e^{-j\omega t} & 0 \\ 0 & \mathbf{I}_L e^{+j\omega t} \end{bmatrix} \quad (12)$$

Estimate of the cyclic-correlation matrix for the extended-data model is given as

$$\mathbf{R}_E = \begin{bmatrix} \mathbf{R}_{xx} & \mathbf{R}_{xx^*} \\ \mathbf{R}_{xx^*}^* & \mathbf{R}_{xx}^* \end{bmatrix} \quad (13)$$

Based on the principle of MUSIC the number of sources that can be calculated by the Extended-cyclic MUSIC algorithm is **K**. So, the number of sources that can be detected is more than the number of sources.

The maxima of the direction of arrivals of the signals of interest(SOI) gives the spatial spectrum that is given by

$$G(\omega) = \frac{1}{c^H(\omega) \mathbf{U}_1 \mathbf{U}_1^H c(\omega) - \|c^T(\omega) \mathbf{U}_2 \mathbf{U}_2^H c(\omega)\|} \quad (14)$$

The above algorithm is a signal selective method with no limitations on finding the direction of arrival of the signals unlike the cyclic MUSIC algorithm.

IV. WAVELET DECOMPOSTION

Wavelet denoising[10-11] has the ability to regain the signal from the noisy environments by concentrating the energy of the signal into smaller values of data coefficients. Wavelet denoising is widely used in signal processing. Out of the many methods available for denoising, temporal denoising is used due to the availability of more number of snapshots. The problem of reconstructing the signal s from the noisy signal n is given by

$$n = s + h \quad (15)$$

where g is the Gaussian noise with zero mean and variance σ^2 .

The Weiner filter is given by $\hat{s} = V \Lambda V^T n$ where V is the Karhunen-Loeve (KL) transform and Λ is the $\text{diag}[\lambda_1/(\lambda_1 + \sigma^2), \dots, \lambda_1/(\lambda_1 + \sigma^2)]$ here λ_1 specifies the i^{th} eigen value of the covariance matrix R_{yy} .

Let M be the orthonormal wavelet matrix, where the denoised signal can be expressed in the wavelet domain as

$$\Phi = \gamma + \eta \quad (16)$$

where $\Phi = Mn$, $\gamma = Ms$ and $\eta = Mg$. Here the mapping of h to η is performed by the orthonormal wavelet transform which has the same specifications of h and compressing the signal into the coefficients in γ . Using a threshold value smaller values of the noise are removed by the wavelet denoising process. The denoising process is a digital filtering process of Φ using the filter

$$G = \text{diag}[g(1), g(2), \dots, g(M)] \quad (17)$$

The parameter G , can be found using the following applied hard and soft threshold [12-14] equations

$$g(j) = \begin{cases} 1 & \text{if } |\Phi(j)| < \tau \\ 0 & \text{elsewhere} \end{cases} \quad (18)$$

and

$$g(j) = \begin{cases} \left(1 - \frac{\tau}{|\Phi(j)|}\right) & \text{if } |\Phi(j)| < \tau \\ 0 & \text{elsewhere} \end{cases} \quad (19)$$

The wavelet domain gives the estimate of the signal using the equation

$$f = N^{-1} G N n \quad (20)$$

The wavelet denoising process decreases the noise power considerably. Also, the non diagonal elements of the noise covariance matrix degrade the Extended-cyclic MUSIC algorithm to some extent. The denoised Extended-cyclic MUSIC algorithm is reviewed by taking M number of snapshots from the signal model. These M snapshots are taken to form the matrix $Q=[q(1),q(2),\dots, q(M)]$. The elements of the matrix are segregated into real and imaginary parts to form the sequences q_i^{real} and q_i^{img} . Denoising is performed using Wavelet decomposition by choosing a suitable threshold for the real q_i^{real} and imaginary q_i^{img} parts respectively for all the sensors. Finally the denoised data matrix \hat{Q} is formed by using the real values and the imaginary values of the denoised sensor arrays. The Extended-cyclic MUSIC spectrum that has the data matrix \hat{Q} is evaluated for computing the direction of arrivals.

V. SIMULATION RESULTS

The simulation results are taken to show performances of Wavelet denoised Extended-cyclic MUSIC. Consider a linear array that is uniformly spaced of size $H=8$ with an element to element spacing of $d = \lambda/2$ which have the same power. The number of snapshots taken for the simulations are $X=250$. Wavelet decomposition is applied for all the individual output sensors independently. The Daubechies wavelet is configured such that it temporally denoises the data matrix. The number of simulations using Monte Carlo are taken to be 500 for each method. The SNR of the output denoised signal is calculated using the equation given by

$$S_o = 10 \log_{10} \left(\frac{E(\|r\|^2)}{E(\|r - \hat{r}\|^2)} \right) \quad (21)$$

where r is the transmitted noiseless signal and \hat{r} is the output denoised signal. The estimate of the variance of noise σ^2 is derived from the coefficients from the first level Wavelet decomposition, where these noises are spatially correlated noises[15-17]. Correlation coefficient calculated between n_1 and n_3 is given by

$$c_{13} = \frac{\text{cov}(n_1, n_3)}{\sqrt{\text{cov}(n_1, n_3) \text{cov}(n_3, n_1)}} \quad (22)$$

where $\text{cov}(n_1, n_3)$ is the covariance of n_1 and n_3 .

Figure.1 gives the simulations where signals from three directions $10^\circ, 25^\circ$ and 45° are taken with high correlation coefficient and low SNR value. The approach utilizes the db22 wavelet for denoising and Figure 1. shows the output SNR S_o that is obtained as a result of the denoising which is a function of the input SNR.

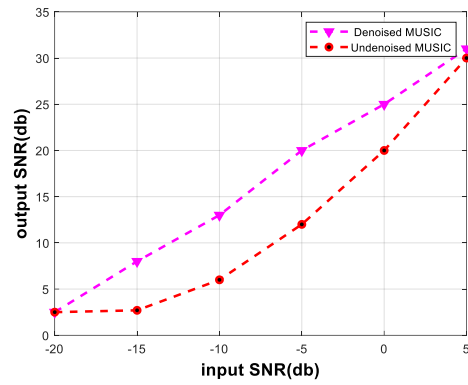


Fig.1 Output SNR vs input SNR for denoised and undenoised Extended cyclic MUSIC

From the above it is evident that the Wavelet denoising technique is very useful for input SNR more than -10dB. The figure shows the simulations of the Extended cyclic MUSIC and the denoised Extended cyclic MUSIC where the results show here that output SNR of proposed method which is the denoised Extended cyclic MUSIC is greater than the Extended cyclic MUSIC algorithm.

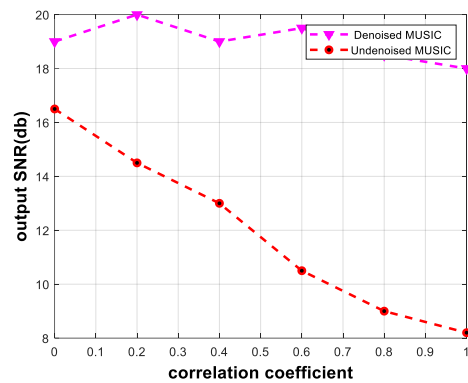


Fig.2 SNR output vs correlation coefficient for denoised and undenoised Extended cyclic MUSIC

Figure.2 shows the simulations where signals from three different closely spaced directions $10^\circ, 20^\circ$ and 45° are taken with high correlation coefficient and low SNR value. The performance estimate of the denoised data is improved because Wavelet decomposition reduces the MSE of the evaluated covariance matrix estimate.

VI. CONCLUSION

In the proposed approach the problem of DOA estimation has been reviewed using the Wavelet decomposition technique for planar waves when spatially correlated noises are present. The Wavelet decomposition technique improves the SNR of the received noisy signal. This technique denoises the noisy signal before estimating the DOA. The enhanced DOA estimation processes the sensors arrays individually in parallel.

Extended cyclic MUSIC is used in the Wavelet decomposition process to reduce the MSE of the estimated signal covariances and hence the direction of arrival estimation performance increases. Simulation values prove here that proposed method is very effective and has good SNR ratios.

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